Eight Annual Front Range Applied Mathematics Student Conference

March 3, 2012

Registration with light refreshments: 8:15 - 9:00

Morning Session I - NC1603
9:00 - 11:00

9:00 - 9:20	Per Sebastian Skardal *University of Colorado, Boulder*	Hierarchical Synchrony in Networks with Community Structure
9:25 - 9:45	Daniel B. Larremore *University of Colorado, Boulder*	Excited About Inhibition - the Surprising Collective Effect of Inhibitory Nodes in Simple Excitable Networks
9:50 - 10:10	Jennifer Diemunsch *University of Colorado, Denver*	Rainbow Matchings in Properly Edge-Colored Graphs
10:15 - 10:35	Nicole Look *University of Colorado, Boulder*	The Nonlinear Dynamics of Running: Symmetry, Stability, and the Effects of Amputation
10:40 - 11:00	Lenton McLendon *University of Colorado, Boulder/RES*	An Improvement of the Measure Corollate Predict Algorithm Using Artificial Neural Networks

Morning Session II - Room NC1605
9:00 - 11:00

9:00 - 9:20	Henricus Bouwmeester *University of Colorado, Denver*	Tiled QR Factorization Algorithms
9:25 - 9:45	Eric Sullivan *University of Colorado, Denver*	A Numerical Model for Vapor Transport and Interface Dynamics in Micro Scale Evaporation
9:50 - 10:10	Jacob Pettine and Nutthavuth Tamang *University of Colorado, Boulder*	Numerical Solution for Small Angles of the Laplace-Young Capillary Equation
10:15 - 10:35	David Appelhans *University of Colorado, Boulder*	Parallel Solution of Elliptic PDEs Using Weighted Adaptive Mesh Refinement with FOSLS-AMG

10:40 - 11:00	Parvaneh Darafshi *University of Colorado, Denver*	The Lanczos Algorithm and Fermat's Factoring Method

Morning Session III - Room NC1607
9:00 - 11:00

9:00 - 9:20	Mark Mueller *University of Colorado, Denver*	One-Dimensional Averaging in Multiphase Systems
9:25 - 9:45	Volodymyr Kondratenko *University of Colorado, Denver*	Ignition from a Fire Perimeter in a WRF Wildland Fire Model
9:50 - 10:10	Lori Ziegelmeier, Michael Kirby, and Chris Peterson *Colorado State University, Fort Collins*	Locally Linear Embedding Clustering with an Application to Landscape Ecology
10:15 - 10:35	Zachary Mullen and Stephen Caan Taylor *University of Colorado, Boulder*	Least Squares Finite-Element Formulation of Non- Newtonian Fluids Exhibiting Shear-thinning and Yield Stress with Potential Applications to Hemorheology
10:40 - 11:00	Kannanut Chamsri *University of Colorado, Denver*	Modeling the Flow of PCL Fluid Due to the Movement of Lung Cilia

Break: 11:00 - 11:15

Plenary Address: 11:15 - 12:15, NC1130

Professor Michael Waterman, *University of Southern California*

Eulerian Graphs and Reading DNA Sequences

Lunch: 12:20 - 1:00

Afternoon Session I - Room NC1603
1:00 - 4:05

1:00 - 1:20	Eric M. Hanson *Colorado State University, Fort Collins*	Using Numerical Algebraic Geometry

1:25 - 1:45	Ryan D. Lewis *University of Colorado, Boulder*	A New Method for Designing Highly Efficient Digital Filters
1:50 - 2:10	Tracy Babb *University of Colorado, Boulder*	Approximation of Power Functions Using Sums of Exponentials
2:15 - 2:25	10 Minute Break	
2:30 - 2:50	Brad Lowery *University of Colorado, Denver*	A New Algorithm for the Collective Communication Operation, Reduce (All-to-One)
2:55 - 3:15	Timothy Morris *University of Colorado, Denver*	Forbidden Subgraphs and Pairs of Disjoint 1-factors
3:20 - 3:40	Michael Steinman *University of Colorado, Colo. Springs*	Periodic Orbits for a Horizontally Driven, Damped and Weakly Nonlinear Pendulum
3:45 - 4:05	Jewell Anne Hartman *University of Colorado, Colo. Springs*	The Frontier Between Mathematics, Physics, and Medicine: Mathematical Techniques for Solving Maxwell Relations for TM Resonance of Surface Plasmons to Aid Biophysics

Afternoon Session II - Room NC1605
1:00 - 4:05

1:00 - 1:20	Danny Dauwe, Taylor Klotz and Chris Karwin *University of Colorado, Colo. Springs*	*Modeling Contest Problem A* Fibonacci Leaves: Finding the Weight of Leaves on a Tree, and Tree Geometry
1:25 - 1:45	Henry Romero *University of Colorado, Boulder*	Enabling Higher Data Rates with Common Messages
1:50 - 2:10	Michael Melcher *University of Colorado, Denver*	Reachability in Arc-Colored Tournaments
2:15 - 2:25	10 Minute Break	
2:30 - 2:50	Jeffrey Larson *University of Colorado, Denver*	Derivative-free Techniques for Minimizing Stochastic Functions
2:55 - 3:15	Ty Thompson *Colorado School of Mines*	An Efficient Spectral Galerkin Method for a Stochastic Ginzburg Landau Model

3:20 - 3:40	Tommaso Buvoli *University of Colorado, Boulder*	Rogue Waves in Optics and Water
3:45 - 4:05	Marshall Carpenter *University of Colorado, Boulder*	Size and Duration of Avalanches in Complex Networks

Plenary Speaker

11:15 - 12:15, NC1130

Eulerian Graphs and Reading DNA Sequences

Michael Waterman

Professor of Biological Sciences, Mathematics, and Computer Science
University of Southern California

With the discovery of the double helix in 1953, it became clear that determining DNA sequences was an important goal. The Sanger experimental method was invented in 1975 and by 2001 refinements of that methods allowed sequencing of the human genome. Today an exciting new generation of sequencing technologies are rapidly increasing the speed of DNA sequencing. This lecture will consider the mathematical and computational challenges of sequencing DNA.

MORNING SESSION I

HIERARCHICAL SYNCHRONY IN NETWORKS WITH COMMUNITY STRUCTURE

Per Sebastian Skardal
University of Colorado, Boulder

Large networks of interacting agents are pervasive in science and nature and serve as important models for emergent collective behavior. Synchronization of such networks is an important area of research with applications in both biology and engineering. In this talk I will discuss synchrony of dynamical systems in heterogeneous networks. I will begin by introducing previous work that combines elements from both the internal dynamics and network structure (e.g. eigenvalues of the adjacency matrix) to describe the synchronization of such networks. However, these results breakdown when network structure becomes modular, i.e. when a network develops many strongly connected communities with weak connections in between. One commonly observed phenomenon in modular network is a hierarchy of synchrony, characterized by distinct communities synchronizing within (i.e. locally) before they synchronize with each other (i.e. globally). I will present a model built to investigate synchronization in networks with many strong communities along with analytical results (supported numerically) that describe the dynamics and hierarchy of synchrony of these systems in addition to other observed phenomena.

EXCITED ABOUT INHIBITION - THE SURPRISING COLLECTIVE EFFECT OF INHIBITORY NODES IN SIMPLE EXCITABLE NETWORKS

Daniel B. Larremore[1]
University of Colorado, Boulder

The neuronal network that makes up the mammalian cortex includes a mix of approximately 80% excitatory and 20% inhibitory neurons. The balance between excitation and inhibition is of interest to theoreticians as well as experimentalists, who have conducted fascinating in-vitro experiments using slices of rat cortex in the presence of various drugs, finding that the cortex appears to operate in state of balanced excitation and inhibition. We explore the dynamics of this balanced state using a simple model for individual excitatory and inhibitory neurons, analyzing the collective dynamics arising when many such neurons are coupled in a complex network. We examine the effect of network structure on dynamics, as well offering insight into in-vitro experimental results. Our surprising preliminary finding is that inhibitory neurons have the indirect effect of guaranteeing ceaseless network activity. This work is a collaboration with Juan G. Restrepo[1], Francesco Sorrentino[2], Woodrow L. Shew[3], and Edward Ott[4].

[2]Department of Mechanical Engineering, University of New Mexico
[3]Department of Physics, University of Arkansas
[4]Institute for Research in Electronics and Applied Physics, University of Maryland

RAINBOW MATCHINGS IN PROPERLY EDGE-COLORED GRAPHS

Jennifer Diemunsch
University of Colorado, Denver

In an edge-colored graph, a rainbow matching is a set of independent edges where each edge has a distinct color. Recently, Wang asked if there is a function $f(\delta)$ such that a properly edge-colored graph G with minimum degree δ and order at least $f(\delta)$ must have a rainbow matching of size δ. This talk responds to this question, answering in the affirmative.

THE NONLINEAR DYNAMICS OF RUNNING: SYMMETRY, STABILITY, AND THE EFFECTS OF AMPUTATION

Nicole Look[1], Elizabeth Bradley, Rodger Kram, Alena Grabowski and William McDermott
University of Colorado, Boulder[1]

The dynamical nature of human locomotion permits the application of nonlinear dynamic analysis to time-series gait data. Conventional analysis methods in biomechanics sufficiently calculate averages, but do not capture subtle features of how the gait pattern changes over time. To quantify dynamic stability, nonlinear dynamic analysis determines how human locomotion resists change after perturbations. Recent application of nonlinear dynamic tools to locomotion has primarily focused on stability during walking. Little research has assessed stability during running using nonlinear dynamics. Furthermore, if nonlinear dynamics can help explain the likelihood of falling while walking, then the nonlinear dynamic analysis of running may help evaluate how asymmetries of runners with transtibial amputations who use prostheses affect stability. Many runners with unilateral transtibial amputations (AMP) report a fear of falling during running and experience more falls than runners with biologically intact legs (NA) (Miller, 2001). Rehabilitation techniques have attempted to increase stability in AMP subjects during walking, specifically by addressing asymmetries between the affected and unaffected leg (Vanicek, 2008). The results of quantifying dynamic stability could potentially be an important tool for identifying people who may be at a greater risk of falling and for evaluating the efficacy of rehabilitation programs from an assessment of asymmetry. We investigated stability during running across a range of speeds and its relationship to leg asymmetry, using analysis, by comparing the legs of AMP runners to the legs of NA runners. The results demonstrated that lower-limb mechanics are less stable for the affected leg of runners with unilateral transtibial amputations compared to the unaffected leg or the control runners. Surprisingly, the results also show that the center of mass mechanics for runners with intact biological legs show slightly lower dynamic stability than runners with unilateral transtibial amputations. Our results suggest that leg asymmetries may lead to leg instability but are compensated for by increased control of center of mass mechanics. For all subjects, stability decreased as speed increased

AN IMPROVEMENT OF THE MEASURE COROLLATE PREDICT ALGORITHM USING ARTIFICIAL NEURAL NETWORKS

Lenton McLendon
University of Colorado, Boulder/Renewable Energy Systems

An introduction to the standard wind industry Measure - Correlate - Predict (MCP) procedure for wind sites is presented. The use of Artificial Neural Networks (ANNs) in the MCP procedure is discussed. The general structure of feed-forward ANNs are introduced. The use of committees of ANNs is presented. The questions of importance of input variables and improvement of prediction are discussed. The question of predictions without wind speed as an input is also investigated. Future topics of interest are discussed as well.

MORNING SESSION II

TILED QR FACTORIZATION ALGORITHMS

Henricus Bouwmeester
University of Colorado, Denver

This work revisits existing algorithms from the 1980's for the QR factorization of rectangular matrices composed of $p \times q$ tiles, where $p \geq q$. We provide a review of the previous algorithms such as the Sameh-Kuck, Fibonacci, and Greedy and present results of novel applications of these algorithms to tile algorithms. Tile algorithms enable high degree of parallelism and low communication requirement. The need for such algorithms is motivated by the apparition of multi-core platforms, but they also prove useful in hybrid computation (CPU+GPU), parallel distributed computation (cluster of nodes with MPI interconnect), or even grid computing.

This is joint work with Julien Langou, Mathias Jacquelin and Yves Robert.

A NUMERICAL MODEL FOR VAPOR TRANSPORT AND INTERFACE DYNAMICS IN MICRO SCALE EVAPORATION

Eric Sullivan
University of Colorado, Denver

Diffusion of water vapor in porous media is a well studied phenomenon that has come under scrutiny in recent experiments. Classical mathematical and empirical models fail to properly capture the dynamics of vapor diffusion in these complex media, and empirical diffusion-enhancement factors have been proposed by many experimentalists to curb these issues. In this presentation we re-examine the micro scale dynamics of evaporation, and propose a new vapor transport model. This new model is then used to propose a Stefan-type model governing the dynamics of moving interfaces in micro scale pores. Numerical solutions to this model are presented along with comparisons to recent experimental work. The goal of this work is to lay the foundation for future models that predict total moisture transport without the use of empirical *diffusion-enhancement* factors.

NUMERICAL SOLUTION FOR SMALL ANGLES OF THE LAPLACE-YOUNG CAPILLARY EQUATION

Jacob Pettine and Nutthavuth Tamang
University of Colorado, Boulder

Capillarity is the physical phenomenon of fluid rise against a vertical wall. The height of the fluid at the wall interface depends upon the geometry of the wall, the wall material and the fluid, and is predicted by the Laplace-Young capillary equation. Of particular interest to industrial semiconductor dipping applications, for example, is the height rise due to wedges of various angles. However, the solution of the Laplace-Young capillary equation for small-angled wedges ($0 < \alpha < \frac{\pi}{2} - \gamma$, where γ is the angle of the contact against a flat wall - dependent upon material and fluid properties - and 2α is the wedge angle) is unbounded as it approaches the corner, and standard finite element methods fail to converge. We propose using the first-order system LL* (FOSLL*) finite element method to determine numerical solutions for the Laplace-Young capillary equations for the small-angle wedge.

PARALLEL SOLUTION OF ELLIPTIC PDES USING WEIGHTED ADAPTIVE MESH REFINEMENT WITH FOSLS-AMG

David Appelhans
University of Colorado, Boulder

This talk presents results on a domain decomposition multigrid method that builds a patchwork global solution from overlapping processor grids that do not communicate during a V-cycle. Typical domain decomposition parallel multigrid implementations exchange information along processor boundaries on each grid level; however, modern supercomputer architecture continues to add more computational power while communication remains expensive. This necessitates a new breed of numerical algorithms that are designed to minimize communication from the start, even at the expense of more on-processor computation. The method presented in this talk leverages the power of ACE adaptive mesh refinement and weighted FOSLS by specifying a weighted functional that emphasizes a processors domain and falls off with distance. By using this weighted functional with ACE, a grid is adaptively generated that focuses on the processor domain while still allocating grid points to areas outside the processor domain where the functional is large and, importantly, extends to the boundary of the global domain. The total number of grid points used by each processor is a small multiple of the grid points within the home domain. The solutions from each processor are then patched into a global approximation and the process can be iterated.

THE LANCZOS ALGORITHM AND FERMAT'S FACTORING METHOD

Parvaneh Darafshi
University of Colorado, Denver

There are different methods for solving systems of linear equations numerically, but the goal of being accurate, quick, and using minimal memory is unified. For matrices that are large and sparse, one of the preferred algorithms is the Lanczos algorithm, an iterative approach. In this talk, the Lanczos algorithm and its applications for solving system of linear equations over a finite and infinite field will be introduced. If the system of the linear equations is over $\mathbb{R}$, the Lanczos algorithm is always successful regardless of rounding problems. The succession and application of the Lanczos algorithm over finite fields towards factoring large integers is in this talk's intentions. Since factoring large integers can be a threat for computers' security, it is important to know how difficult it would be for the intruders to do the factoring of the computer encoding password number. When the application of Lanczos algorithm over finite fields meets Fermat's square pair factorization method, we can discuss this difficulty.

MORNING SESSION III

ONE-DIMENSIONAL AVERAGING IN MULTIPHASE SYSTEMS

Mark Mueller
University of Colorado, Denver

A multiphase material has one or more components, such as clay and water, in at least two phases, such as liquid water in the solid clay. In this talk we consider two scale sizes. In the first scale, a "microscale", the solid and liquid phases can be distinguished from one another. The phases, and their interactions, are handled in detail. In the second scale, a "macroscale", the phases are not distinguished and the water and clay, for example, are handled together. Experimental results are often available on a macroscale, while basic principles, such as balance laws, often hold at the microscale. We will look at a simple example of an averaging process, in which field equations of the material at the microscale are averaged to get macroscale variables. The goal is to understand how microscale properties relate to these macroscale variables.

IGNITION FROM A FIRE PERIMETER IN A WRF WILDLAND FIRE MODEL

Volodymyr Kondratenko
University of Colorado, Denver

The current wildland fire model starts the fire from a given ignition point at a given time. We want to start the model from a given fire perimeter at a given time instead. However, the fuel balance and the state of the atmosphere depend on the history of the fire. The purpose of this work is to create an approximate artificial history of the fire based on the given fire perimeter and time and an approximate ignition point and time. Replaying the fire history then establishes a reasonable fuel balance and outputs heat fluxes into the atmospheric model, which allow the atmospheric circulation to develop. Then the coupled atmosphere-fire model takes over. In this preliminary investigation, the ignition times in the fire area are calculated based on the distance from the ignition point to the perimeter, assuming that the perimeter is convex or star-shaped. Simulation results for an ideal example show that the fire can continue in a natural way from the perimeter. Possible extensions include algorithms for more general perimeters and running the fire model backwards in time from the perimeter to create a more realistic history.

This talk will be based on the results of the paper:
V. Y. Kondratenko, J. D. Beezley, A. K. Kochanski, J. Mandel, Ignition from a fire perimeter

in a WRF wildland fire model, Paper 9.6, 12th WRF Users Workshop, National Center for Atmospheric Research, June 20-24, 2011.

LOCALLY LINEAR EMBEDDING CLUSTERING WITH AN APPLICATION TO LANDSCAPE ECOLOGY

Lori Ziegelmeier, Michael Kirby, and Chris Peterson
Colorado State University, Fort Collins

The ability to characterize the color content of natural imagery is an important application of image processing. The pixel by pixel coloring of images may be viewed naturally as points in color space, and the inherent structure and distribution of these points affords a quantization, through clustering, of the color information in the image. We present a novel topologically driven clustering algorithm that permits segmentation of the color features in a digital image. The algorithm blends Locally Linear Embedding (LLE) and vector quantization by mapping color information to a lower dimensional space, identifying distinct color regions, and classifying pixels together based on both a proximity measure and color content. It is observed that these techniques permit a significant reduction in color resolution while maintaining the visually important features of images.

LEAST SQUARES FINITE-ELEMENT FORMULATION OF NON-NEWTONIAN FLUIDS EXHIBITING SHEAR-THINNING AND YIELD STRESS WITH POTENTIAL APPLICATIONS TO HEMORHEOLOGY

Zachary Mullen and Stephen Caan Taylor
University of Colorado, Boulder

We discuss the underlying theory and preliminary results for a least-squares finite element formulation of blood flow PDEs. Modeling blood as a non-Newtonian fluid exhibiting both properties of shear-thinning and yield stress, the Navier-Stokes equations are generalized by the Casson constitutive equation to describe the physical problem. With use of Newton's method to linearize the model, the finite element least-squares formulation can be expressed as a strictly positive definite linear system.

We report progress on current research utilizing a 4-1 contraction problem to investigate behavior in irregular domains, including areas of stagnation and backflow which may match pathological conditions. Challenges include integration of time-dependent or pulsatile flow to more closely match the physical problem and the existences of boundary singularities. Potential applications include examination of blood flow in vessel anomalies as well as thrombus development.

MODELING THE FLOW OF PCL FLUID DUE TO THE MOVEMENT OF LUNG CILIA

Kannanut Chamsri
University of Colorado, Denver

Cilia in the human lungs are moving hairs that aid in the movement of mucus. The layer that contains the cilia is called the periciliary layer, PCL, and the liquid in that layer the PCL fluid. We consider the PCL fluid as an incompressible viscous fluid. For an initial model of the fluid flow in the PCL, we consider the overall height as a constant, the portion of the PCL with cilia as a porous medium, and the PCL fluid above the cilia as undergoing Stokes flow. To calculate the fluid velocity in the PCL layer, Stokes-Brinkman equations are applied with a fixed boundary height of the PCL layer. The numerical result is compared with an analytical solution when the top boundary fluid is moving at a constant velocity in order to validate the numerical solution.

AFTERNOON SESSION I

USING NUMERICAL ALGEBRAIC GEOMETRY

Eric M. Hanson
Colorado State University, Fort Collins

Numerical Algebraic Geometry is a relatively young field of mathematics, where homotopy continuation is applied to the problem of finding roots to large systems of polynomial equations. A brief introduction to this field and its applications will be discussed. Specifically, techniques of Numerical Algebraic Geometry and fiber products can be applied in the field of Kinematics to find mechanisms with greater than expected mobility.

A NEW METHOD FOR DESIGNING HIGHLY EFFICIENT DIGITAL FILTERS

Ryan D. Lewis
University of Colorado, Boulder

Digital filters are ubiquitous, appearing in virtually all modern electronic devices and utilized in many software-based data processing systems. Frequently, the capabilities of a system are limited by the computational cost of its digital filters, since a higher computational cost demands a combination of more power, circuitry, and processing time. Thus, there is great interest in constructing computationally efficient digital filters.

From a mathematical perspective, the problem of designing a digital filter amounts to solving an approximation problem using polynomials or rational functions, subject to certain additional constraints that ensure the resulting filter is stable. Effective algorithms exist for solving this approximation problem with polynomials, but the resulting filters are far less efficient than what could be obtained with rational functions. On the other hand, effective algorithms for solving the approximation problem with rational functions have remained elusive, in part because the approximation problem is further complicated by the stability constraints.

In this talk, I will present a new method for designing highly efficient digital filters. Our method begins with a critical observation: an unstable filter may be converted into a stable filter, thus rendering the stability constraints that complicate the rational approximation problem unnecessary. Next, we use recent advances in rational approximation to obtain near-optimal rational filters. The combination of these techniques leads to a robust, nearly automatic method for filter design. After providing a brief introduction to digital filtering, I will present our new method and provide examples of the filters it produces. This work is in collaboration with Gregory Beylkin (beylkin@colorado.edu) and Lucas Monzón (lucas.monzon@colorado.edu) with the Department of Applied Mathematics, University of Colorado, Boulder.

APPROXIMATION OF POWER FUNCTIONS USING SUMS OF EXPONENTIALS

Tracy Babb
University of Colorado, Boulder

A change of variables in the gamma function can be done to find a formula for power functions in terms of an integral of an exponential function. The trapezoidal rule can be used to find an approximation for the power function in terms of a sum of exponentials. The approximation is desired on $[1, \infty]$, with an initial shift to find the approximation on $[1-\delta, \infty]$. The initial shift results in less information being lost at x=1 when the final result is shifted back. It can be shown that for any $\epsilon > 0$ the parameters for the trapezoidal rule can be chosen such that the error is less than ϵ. While this method provides a reasonable approximation, it uses a suboptimal number of terms. A reduction algorithm is then used to decrease the number of terms used while retaining an error bound of ϵ_2 for some $\epsilon_2 > \epsilon$. Results are found for a desired error in the range $[10^{-4}, 10^{-16}]$. Finally, these tabulated results are compared against the known theoretical bound on the error for the optimal approximation. Results are also compared with both the theoretical and actual results obtained by Braess and Hackbusch. Applications of the approximation exist in quantum chemistry, quantum physics, convolution integrals, wavelet applications, and many other fields. Finally, the theory in this algorithm can also be used to find an approximation of rational functions.

A NEW ALGORITHM FOR THE COLLECTIVE COMMUNICATION OPERATION, REDUCE (ALL-TO-ONE)

Brad Lowery
University of Colorado, Denver

Applications performed on parallel computers require information to be shared between processors. The operations that communicate information between a collection of processors are called a collective communication. Reduce (All-to-One) takes a message on each of the processors, combines the messages entry-wise, and returns the value on one specified processor. We introduce a new algorithm for the reduction operation, the greedy algorithm.

For short messages, it is know that a binomial algorithm is optimal. But, for long messages the message can be segmented and it is beneficial to use an algorithm that allows processors to work simultaneously on separate segments. The question we want to answer is when given a message of size m on p processors, and machine parameters, which algorithm should be chosen? When comparing the greedy algorithm to standard reduction algorithms for various values of the parameters it is seen that, theoretically, the greedy algorithm is the best.

The greedy algorithm is implemented using the software MPI (message passing interface) and compared to MPI's function MPI_Reduce as well as implementations of standard reduction algorithms. Numerical results are discussed and compared with the theoretical results.

Finally, we investigated the effect of using segments of unequal size, a topic previously not given much attention. For the greedy algorithm, unequal segmentation showed some improvement over equal segmentation.

FORBIDDEN SUBGRAPHS AND PAIRS OF DISJOINT 1-FACTORS

Timothy Morris
University of Colorado, Denver

A graph G is said to have a k-factor if G contains a k-regular subgraph that spans all the vertices of G. G is said to be *Hamiltonian* if G contains a cycle using all the vertices of G. Thus, if G is Hamiltonian, then G contains a 2-factor. However, the converse is not necessarily true. For an even graph G, if G is Hamiltonian G also contains disjoint 1-factors. Any graph that contains disjoint 1-factors contains a 2-factor, but the converse does not hold. Thus the property contains disjoint 1-factors lies between contains a 2-factor and Hamiltonian.

Given a family $\mathcal{F}$ of graphs we say that G is $\mathcal{F}$-free if G does not contain a copy of any graph in $\mathcal{F}$ as an induced subgraph. Various conditions, including forbidden subgraph conditions, have been studied to guarantee that a graph is Hamiltonian as well as for the property contains a 2-factor. We consider a condition that is sufficient to guarantee that a graph has a 2-factor, but not that it is Hamiltonian, and determine what this condition can tell us about the property contains disjoint 1-factors. In particular, we show that if an even graph G is 2-connected and $\{K_{1,4}, P_4\}$-free, then G either contains disjoint 1-factors or falls into a small infinite family of graphs.

PERIODIC ORBITS FOR A HORIZONTALLY DRIVEN, DAMPED AND WEAKLY NONLINEAR PENDULUM

Michael Steinman
University of Colorado, Colorado Springs

A method for finding approximate periodic orbits of a damped and horizontally driven, weakly nonlinear oscillator is presented. The first step is to consider an undamped, free Hamiltonian system. Viewing the damped free system as a perturbation of the Hamiltonian system, approximate solutions to the damped, free oscillator are derived using the method of averaging. Subsequently, a variational method is used to treat the damped, driven regime, which is considered to be a perturbation of the damped, free system. Finally, using a resonance condition, the Melnikov functions are defined, the zeros of which correspond to approximate periodic orbits of the damped and driven oscillator.

THE FRONTIER BETWEEN MATHEMATICS, PHYSICS, AND MEDICINE: MATHEMATICAL TECHNIQUES FOR SOLVING MAXWELL RELATIONS FOR TM RESONANCE OF SURFACE PLASMONS TO AID BIOPHYSICS

Jewell Anne Hartman
University of Colorado, Colorado Springs

The frontier that exists between biology/medicine and physics, known as biophysics, requires sophisticated tools and techniques of applied mathematics to aid its discoveries. The fundamental questions of biophysics involve matrices, dispersion relations, differential equations (ODEs and PDEs, linear and nonlinear), in addition to dispersion relations and boundary conditionsall situations which require advanced applied mathematics.

Metamaterials are materials whose fundamental electric and/or magnetic qualities (permittivity and permeability, respectively) are artificially dictated. As a result, there exist three-fourths more materials than previously known. The properties of these materials and consequently, their applications, have remained largely unknown until recently. In order to discover their properties, Maxwell relations and Fresnel coefficients must be re-solved. This requires strong application of varying techniques in applied mathematics. Non-linearity arises in many situations, further complicating the equations and requiring the applied mathematics. Additionally, dispersion relations need to be solved in order to explain the plasmon resonance occurring at the boundary between a metal (usually a nanoparticle) and a dielectric. Consequently, highly non-linear systems must be solved in order to attain the goal.

The ultimate goal of the UCCS BioFrontiers Institute Research Program is cancer treatment and prevention through the discoveries of these nanotechnologies and SPP (surface plasmon polariton). However, without the applied mathematics, none of this would be possible. This presentation provides an overview of the research involved in the BioFrontiers program and the mathematics involved in accomplishing these discoveries. Techniques of solving these dispersion relations, systems of equations, matrices, and differential equations combine applied mathematics and mathematical programs, which will involve MATLAB, Mathematica, and FORTRAN. Therefore, in order to succeed in accomplishing the future of medicine as the frontier between physics and biology is discovered, applied mathematics will serve as the tool that makes it all possible.

AFTERNOON SESSION II

Modeling Contest, Problem A
FIBONACCI LEAVES: FINDING THE WEIGHT OF LEAVES ON A TREE, AND TREE GEOMETRY

Danny Dauwe, Taylor Klotz and Chris Karwin
University of Colorado, Colorado Springs

When observing the structure of a tree one of the first thoughts that comes to mind is the complexity that characterizes the tree. Different types of tree species can have different tree shapes, leaf shapes, and branching structures, but they all constrain themselves to these shapes for the reason of maximizing certain tree functions such as sunlight absorption.

We created mathematical models to help shed light on these intricacies. Our first model was designed to answer the simple question of leaf weight; in this model we make a simple assumption that leaf weight is dependent on the leaf weight of the previous leaf, and from this we generate a simple formula to calculate the weight of leaves on any given branch. This model also briefly explores the geometry of leaf shapes, and leaf placement, which are often connected to the Fibonacci sequence. In our second model, we consider the general structure of the tree and how this structure helps one to estimate the number of leaf branches on a tree, which helps estimate the number of leaves on a tree and consequently the total leaf weight. The basis for the second model is again related to the Fibonacci sequence. The third model asks the question of how the general tree profile can be determined, and after some simple assumptions we are able to create a constrained optimization problem, the results of which determine the geometry of the tree structure.

ENABLING HIGHER DATA RATES WITH COMMON MESSAGES

Henry Romero
University of Colorado, Boulder

Information Theory aims to define the realm of what is and what is not possible in the electrical engineering problem of communication. Past successes in Information Theory have guided the development of efficient codes that enable current wireless data rate standards.

These past successes were primarily in the context of a single transmitter communicating to a single receiver. Current challenges are with multiple transmitters and receivers. We present developments in the Interference Channel, where there are exactly two transmitters and receivers and where recent work has shown that approximate answers can be had where exact answers are elusive. This allows inference into the design of communication systems. One such lesson learned is that if both transmitters share a common message, then per transmitter data rate could be much larger the data rate achievable with no common message.

REACHABILITY IN ARC-COLORED TOURNAMENTS

Michael Melcher
University of Colorado, Denver

A monochromatic sink in an arc-colored tournament is a vertex that can be reached by every other vertex in the tournament by a monochromatic path. A rainbow k-cycle in an arc-colored tournament is a cycle with k arcs such that no two arcs in the cycle have the same color. Motivated by a conjecture of P. Erdős and a conjecture by Sands, Sauer, and Woodrow, we investigate the existence of monochromatic sinks in certain tournaments without rainbow 3-cycles; the tournaments considered are tournaments obtained from special arc reversals of transitive tournaments. Among the tournaments investigated are upset tournaments, tournaments that are obtained by reversing the arcs in a path from the source to the sink in a transitive tournament. Results from the literature and some open problems will also be discussed.

DERIVATIVE-FREE TECHNIQUES FOR MINIMIZING STOCHASTIC FUNCTIONS

Jeffrey Larson
University of Colorado, Denver

Classical techniques for optimization can break down when dealing with noisy functions. Nevertheless, such functions occur in countless applications, a few of which will be highlighted in this talk. We also present an overview of existing techniques for optimizing these stochastic functions, discussing the strengths and weaknesses of these algorithms. We lastly propose a new method for minimizing functions with uncertainty which should preserve theoretical results while improving the performance of practical implementations.

AN EFFICIENT SPECTRAL GALERKIN METHOD FOR A STOCHASTIC GINZBURG LANDAU MODEL

Ty Thompson
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The Ginzburg Landau (GL) model of superconductivity is a classic example of success in using a phenomenological approach when studying systems whose underlying theory is either complex or incomplete. Its predictions of vortex configurations in superconductors have been theoretically justified, numerically verified, and physically observed in many contexts, however, less is understood about their evolution from typical initial states. The Langevin extension of the GL equations offers a natural phenomenological context for the modeling of such dynamics in symmetric superconductors, and while its simulation has traditionally been computationally prohibitive, the ever increasing availability of large scale computing power motivates us to reexamine its utility. With the goal of mitigating the computational challenges of large stochastic dimension, we use a generalized spectral decomposition (GSD) in a fully discrete Galerkin method for a stochastic GL model on thin, cylindrically symmetric superconductors. Our implementation relies upon highly efficient computational algorithms for the deterministic version of the problem, and supports intuitive perspectives of vortex nucleation and motion associated with spatial symmetry breaking.

ROGUE WAVES IN OPTICS AND WATER

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In recent years, large amplitude "rogue" waves have been studied in water waves and nonlinear optics. These large waves occur more frequently than suggested by conventional linear models and there is widespread belief that nonlinear phenomena are responsible for such large waves.

It is widely known that modulational instability causes solutions of the linearized nonlinear Schrdinger equation to experience exponential growth. We extend this result by performing stability analysis on several NLS type equations used in the study of light waves. We determine explicit formulas describing growth rates and mode growth. To confirm the validity of our analytical formulas, we run numerical experiments and look for growth in each mode. We conclude by numerically verifying that rogue waves occur in several of these equations.

SIZE AND DURATION OF AVALANCHES IN COMPLEX NETWORKS

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An excitable network can be expressed by a network of nodes, where the connection between nodes represents the probability an excitation passes from one node to another. Cascades of excitation on such networks have been used to model epidemics, social interactions, gene expression, and neural networks. These cascades (avalanches) have been observed in the brains of awake monkeys and in slices of rat cortex, and are thus of particular biological interest.

The dynamics of the network are related to the Perron-Frobenius eigenvector and eigenvalue of the adjacency matrix. We are most interested in the critical shift which occurs when the largest eigenvalue is one. Not only is this the threshold for avalanches of infinite size, this regime is also the most characteristic of biological systems. The distribution of the size and duration of avalanches can be calculated based on the largest eigenvalue, and the corresponding entry in the Perron-Frobenius eigenvector.