Seventh Annual Front Range Applied Mathematics Student Conference

March 5, 2011

Breakfast and Registration: 8:30 - 8:55

Morning Session I - NC1603 9:00 - 11:00

9:00 - 9:20	Theodore Galanthay *University of Colorado, Boulder*	Why Ignoring Your Darwinian Fitness May Be Adaptive: Evolutionary Dynamics of Movement Strategies in the Presence of Realistic Constraints
9:25 - 9:45	Thomas Trantow *University of Colorado, Boulder*	A Discrete Age Class SIR Model and Its Effects on Modeling an Epidemic
9:50 - 10:10	Jerome Perkins *University of Colorado, Boulder*	Influence of Food Web Structure on Predator-Prey Dynamics in a Patchy Environment
10:15 - 10:35	Marshall Carpenter *University of Colorado, Boulder*	Distribution of Avalanches in Critical Networks
10:40 - 11:00	Joseph Schneiderwind *Colorado School of Mines*	Normal Mode Solution of the Elastic Pekeris Waveguide Problem with Comparisons Against Laboratory Data

Morning Session II - Room NC1605 9:00 - 11:00

9:00 - 9:20	William McCollom *Colorado School of Mines*	A Scaling Method to Construct Lax Pairs
9:25 - 9:45	Jake Rezac *Colorado School of Mines*	Construction of Conservation Laws From Lax Pairs
9:50 - 10:10	Daniel Kaslovsky *University of Colorado, Boulder*	Geometric Image Processing
10:15 - 10:35	Richard M. Charles *University of Colorado, Boulder*	Nonnegative Matrix Factorization: *Image Basis Construction Using an Overlapping Patch Space Approach With NMF*

10:40 - 11:00	Joshua Garland *University of Colorado, Boulder*	Prediction in Projection: Computer Performance Forecasting, a Dynamical Systems Approach

Morning Session III - Room NC1607
9:00 - 11:00

9:00 - 9:20	Jewel Anne Hartman *University of Colorado, Colo. Springs*	Scattered Wave Amplitude and Fourier Analysis of Brillouin Zones for Solid-State Physics
9:25 - 9:45	Stephen Kissler *University of Colorado, Boulder*	Slime Molds and Public Transit
9:50 - 10:10	Jerrad Hampton *University of Colorado, Boulder*	Estimation of Distribution Overlap for Urn Models
10:15 - 10:35	Shilpa Dasgupta *University of Colorado, Denver*	Characterization of 2-Tree Unit Probe Interval Graphs
10:40 - 11:00	Kyla Maletsky *University of Colorado, Boulder*	Adaptive Coupling in Kuramoto Networks of Oscillators

Break: 11:00 - 11:15

Plenary Address: 11:15 - 12:15, NC 1130

Dr. Edward Ott, *University of Maryland*
Collective Behavior in Large Systems of Coupled Dynamical Units

Lunch: 12:15 - 1:00

Afternoon Session I - Room NC1603
1:00 - 3:25

1:00 - 1:20	Monica Grigg *Colorado School of Mines*	G-lattices of an Unrooted Perfect Phylogeny
1:25 - 1:45	Daniel Larremore *University of Colorado, Boulder*	Predicting Criticality and Dynamic Range in Excitable Complex Networks: Effects of Topology

1:50 - 2:10	Tommy McDowell *University of Colorado, Colo. Springs*	Classification of the KP Equation and the Cyclic Shift
2:15 - 2:25	10 Minute Break	
2:30 - 2:50	Timothy Morris *University of Colorado, Denver*	Pancyclicity of 4-connected, Claw-Free, P_{10}-Free Graphs
2:55 - 3:15	Eric Sullivan *University of Colorado, Denver*	Total Differentials, Thermodynamics and Porous Media
3:20 - 3:40	Ty Thompson *Colorado School of Mines*	Theory and Simulation of Critical Dynamics in Ginzburg Landau Models

Afternoon Session II - Room NC1605
1:00 - 3:25

1:00 - 1:20	Prince Agyeman, Monica Blomker, Priscus Kahangwa *University of Colorado, Denver*	*Modeling Contest, Problem C* Analyzing the Economic and Environmental Impact of Widespread Electric Vehicle Use
1:25 - 1:45	Marshall Carpenter, Keegan Boyle, Vicky Li *University of Colorado, Boulder*	*Modeling Contest, Problem A* Air with a Twist: Finding the Optimal Snowboard Half-Pipe
1:50 - 2:10	Justin Baacke, Maxwell Bohning, Andrew Erickson, David Kozak, Patrick O'Brien *University of Colorado, Boulder*	Modeling Ant Colony Behavior
2:15 - 2:25	10 Minute Break	
2:30 - 2:50	Jeremy Garcia *University of Northern Colorado*	Positive Extensions of Matrices Indexed by a Homogeneous Tree
2:55 - 3:15	Scott Pawlowski *University of Colorado, Boulder*	Synchronization in Van der Pol Oscillators
3:20 - 3:40	Kannanut Chamsri *University of Colorado, Denver*	Permeability of Fluid Flow through Cilia

Plenary Speaker

11:15 - 12:15, NC 1130

Collective Behavior in Large Systems of Coupled Dynamical Units
Professor Edward Ott, University of Maryland

An important issue arising in many different situations is the question of how global macroscopic behavior emerges from the coupling of many, typically nonidentical, dynamically evolving subsystems. For example, what is the mechanism by which the rhythmic oscillation of the of order 10,000 individual pacemaker cells in the heart organize themselves to beat in phase with each other? Other examples include synchronous flashing of fireflies, oscillation of electrical activity in the brain, circadian rhythm, coupled lasers, and many others. This talk will give an overview on recent progress in this field, focusing on modeling and mathematical analysis.

About the speaker: Edward Ott earned his bachelor's degree from Cooper Union in the field of electrical engineering. He received his master's and doctoral degrees in electrophysics from Polytechnic Institute of Brooklyn. After a postdoctoral year at Cambridge University, he became a professor of electrical engineering at Cornell University. In 1979, Professor Ott became a faculty member of the University of Maryland at College Park. His areas of research include chaotic scattering, fractals, and control of chaos. Professor Ott has numerous publications and is the author of the book, Chaos in Dynamical Systems, an introduction to concepts in chaotic dynamics for scientists and students.

MORNING SESSION I

WHY IGNORING YOUR DARWINIAN FITNESS MAY BE ADAPTIVE: EVOLUTIONARY DYNAMICS OF MOVEMENT STRATEGIES IN THE PRESENCE OF REALISTIC CONSTRAINTS

Theodore Galanthay
University of Colorado, Boulder

In most theoretical habitat selection models, scientists assume organisms move to maximize their fitness. However, it remains unclear what type of movement mechanisms might produce higher fitness. We analyze a mechanistic single-species two-patch habitat selection model and compute analytically a movement strategy that demonstrates that higher fitness is not achieved by relying upon fitness-dependent cues. We apply the tools of adaptive dynamics to show numerically that this strategy is an ESS and also convergence stable.

A DISCRETE AGE CLASS SIR MODEL AND ITS EFFECTS ON MODELING AN EPIDEMIC

Thomas Trantow
University of Colorado, Boulder

Understanding the dynamics of infections and their host populations is a fundamental goal of epidemiology. Host population structure, arising from various sources of heterogeneity among hosts, has the potential to alter these dynamics in a number of ways. Accordingly, I investigated the effects of a natural characteristic that structures all real populations: age. Using the well-established Susceptible-Infected-Recovered (SIR) modeling framework, I compared predicted dynamics of infections and host populations in the presence and absence of age structure. I will discuss how age structure can significantly affect the theoretical infection dynamics of populations.

INFLUENCE OF FOOD WEB STRUCTURE ON PREDATOR-PREY DYNAMICS IN A PATCHY ENVIRONMENT

Jerome Perkins
University of Colorado, Boulder

A great deal of previous research has sought to predict how spatial distributions of predators are shaped by distributions of prey and the resources prey consume (which the predator does not consume). A common prediction from many different models is known as leapfrogging, where the predators distribution matches that of the preys resource instead of matching the preys distribution. This prediction has arisen from models considering simple three-species chains (resource-prey-predator), yet in nature, communities are webs with multiple species of predators, prey, and resources. I examined whether leapfrogging is predicted in systems with more than three species. In particular, I focused on systems with 5 species: one resource species, two prey species, and two predator species (but still only three trophic levels). My results show some conditions in which the leapfrogging prediction does indeed generalize to communities with more than 3 species, but also yields empirically testable predictions about when leapfrogging is not expected.

DISTRIBUTION OF AVALANCHES IN CRITICAL NETWORKS

Marshall Carpenter
University of Colorado, Boulder

We study the influence of network structure on the properties of avalanches of excitation cascading over the nodes of the network. Such excitable networks have been used to model epidemics, social interactions, and neuronal networks, and avalanches of activity have been observed in real mammalian brains, and are thus of particular interest. While the statistics of avalanches has been studied in random networks, the effect of complex network structure on avalanche properties has not been previously addressed. We begin by deriving the distributions for avalanche size and duration in a purely branched network.

Under reasonable assumptions, we can extend this simple case to approximate avalanche distributions in directed networks. We relate the dynamics to the adjacency matrix determined by the connections in the network. We are most interested in critical networks which occur when the largest eigenvalue of the adjacency matrix is 1. For these networks, we find that the distribution of avalanche sizes and durations is a power law, and that the expected size of an avalanche starting at a given node is proportional to the corresponding entry of the eigenvector associated to eigenvalue 1. Avalanches on such networks are modeled numerically and the distributions of avalanche size and duration are studied to find the level of agreement between our theory and simulations.

NORMAL MODE SOLUTION OF THE ELASTIC PEKERIS WAVEGUIDE PROBLEM WITH COMPARISONS AGAINST LABORATORY DATA

Joseph Schneiderwind
Colorado School of Mines

Research in underwater acoustics is an active area of study with applications ranging from naval to oil exploration to marine mammal studies. Many underwater sound models are based on limiting approximations or are strict numerical solutions to the wave equation. The method presented in this work derives an analytic solution for underwater acoustic transmissions using an integral transform technique. Specifically, acoustic transmissions are simulated by modeling a compressional wave acoustic source within a liquid layer independent of horizontal coordinates. The liquid layer has a pressure-release boundary above, and an elastic solid semi-infinite halfspace below. Normal-mode solutions, based on integral transformation techniques, are implemented with numerical software to simulate amplitude loss of transmitted sound waves. Solutions are compared to a normal-mode solution of the Pekeris waveguide [C.L. Pekeris, Geological Society of America, Memoir 27 (1948)], which assumes a fluid bottom of greater density to approximate the ocean bottom. A fluid bottom eliminates shear wave contributions and so simplifies continuity conditions in modeling. Further, as the shear modulus of an elastic bottom approaches zero, the medium takes the characteristics of a fluid. In order to make comparisons with the Pekeris waveguide, taking a low shear speed in the elastic solid bottom layer should resemble compressional wave transmission loss experienced by the fluid bottom layer case. These normal-mode solutions are benchmarked against experimental data collected at the U.S. Naval Research Laboratory in Washington D.C. The experiment took hydrophone measurements at constant depth and varying distance from a stationary sound source in a water tank, creating virtual array data. The tank contained a suspended, nearly flat, polyvinyl chloride slab that was used to represent an elastic seafloor. Also presented in this work is a discussion of solution dependence on acoustic horizontal wavenumbers and the difficulties in obtaining them.

MORNING SESSION II

A SCALING METHOD TO CONSTRUCT LAX PAIRS

William McCollom
Colorado School of Mines

It is well known that there is no general solution method for non-linear partial differential equations (PDEs). However, an important subset of non-linear PDEs, known as completely integrable PDEs, can be solved by the Inverse Scattering Transform. This transform requires the existence of a Lax pair, which is a pair of differential operators that allows replacement of the non-linear PDE with a system of compatible linear PDEs. In addition to the operator form, there also exists a matrix representation of the Lax pair which is called the zero-curvature representation of the original PDE. In this talk we will discuss the compatibility conditions for the differential operator and matrix formalisms as well as how to convert between the two forms of the Lax pair. Additionally, a method for construction of Lax Pairs by scaling is presented.

CONSTRUCTION OF CONSERVATION LAWS FROM LAX PAIRS

Jake Rezac
Colorado School of Mines

The analytic solvability of nonlinear partial differential equations (PDEs) has been a subject of intensive study since the 1970s. One solution technique developed for such PDEs is the Inverse Scattering Transform (IST), analogous to a Fourier transform. The IST relies heavily on the existence of a Lax pair, i.e., a pair of linear PDEs related to the original nonlinear PDE through a compatibility condition. Also important in the study of integrability of nonlinear PDEs are conservation laws. Both the existence of a Lax pair and an infinite number of conservation laws are indicative of the complete integrability of a nonlinear PDE. In this talk, we will discuss a technique for constructing conservation laws from Lax pairs, based on a method originally developed by Drinfeld and Sokolov.

GEOMETRIC IMAGE PROCESSING

Daniel N. Kaslovsky
University of Colorado, Bouder

Many modern image processing algorithms operate at the scale of image patches rather than the scale of the image as a whole or its individual pixels. The collection of all such patches extracted from an image forms a high dimensional data set but organizes along a low dimensional geometric structure. Efficient image processing may be achieved by learning and exploiting the geometry inherent in such sets. In this talk, I will discuss geometric approaches to image processing and introduce our current research on efficient processing of image patches through local geometric representations.

NONNEGATIVE MATRIX FACTORIZATION: *IMAGE BASIS CONSTRUCTION USING AN OVERLAPPING PATCH SPACE APPROACH WITH NMF*

Richard M. Charles
University of Colorado, Boulder

Matrix factorization methods are well established when the matrix is arbitrary. Nonnegative matrix factorization is a relatively new technique for iteratively factoring matrices. The technique takes a matrix V and ideally factors it into a product of WH, where W is the basis matrix and H is a matrix of weights. As both factors have only non-negative entries, the perfect method amounts to defining a cone in the positive orthant of a high dimensional space for which vectors of objects of interest, e.g. images, are defined. A natural question is: Can this iterative method produce unique factors and under what conditions? The algorithm was originally used on a library of facial images. Our presentation investigates the creation of both a basis for libraries of images and for a single image. We use a "patch space" approach that generates approximations of a specified rank to original images and explore possible future research directions.

PREDICTION IN PROJECTION: COMPUTER PERFORMANCE FORECASTING, A DYNAMICAL SYSTEMS APPROACH

Joshua Garland
University of Colorado, Boulder

Time-series data measuring processor load during the execution of a simple nested loop provides strong evidence to support the claim that computer systems are dynamical systems. Building on this framework, we use a custom measurement infrastructure, delay-coordinate embedding and nonlinear time-series analysis to forecast processor load as well as cache performance. The success of this type of forecasting is in stark contrast to the computer systems community's view of a computer system, which asserts that today's computer systems are a highly complex coupling of several different unobservable stochastic systems. The success of forecasting results based in the theory & methods of deterministic dynamics provides further evidence to support that computer systems are deterministic nonlinear dynamical systems.

The current view in the nonlinear dynamics community with regard to delay coordinate embedding for prediction purposes is that a one to one delay map F is needed. If the objective is to use F(A) to predict the future behavior of trajectories, then it is sufficient to have the map F be one-to-one, in which case the dimension of the reconstructed state space must be at least twice the box counting dimension of the original attractor [Sauer et al.] While this is a sufficient condition, it may not be necessary for the purposes of prediction. A key aspect of this research project is to study the validity of low dimensional projections as a means to automate delay coordinate embedding for forecast schemes – that is, choosing "arbitrary" low (2 or 3) dimensional projections of the reconstructed state space as a way of sampling the temporal flow without worrying about topological conjugacies. The results presented at this talk suggest ways of improving computer design at a systems level as well as evidence to support the use of semi-conjugacies in forecasting schemes. This talk will focus on not only the results of this research but also on the applications to computer design.

MORNING SESSION III

SCATTERED WAVE AMPLITUDE AND FOURIER ANALYSIS OF BRILLOUIN ZONES FOR SOLID-STATE PHYSICS

Jewell Anne Hartman
University of Colorado, Colorado Springs

In today's fast-paced technological world, some of the fastest and latest technological achievements are made possible due to the advancements in Solid State Physics. In order to fully understand the physics necessary to continue the improvement and advancement of technology, complete comprehension of the underlying mathematics is foundational. Important mathematical principles are utilized in the study of wave diffraction, reciprocal lattice, and Brillouin Zones in Solid-State Physics. Fourier transforms are utilized for analyzing the basis and determining atomic form fact, structure factor, and bcc lattice. Vector calculus is also foundational, and the formulas for volumes of Brillouin zones can be calculated thanks to the amazing properties of vector calculus. Hamiltonians of Van der Waals London Interaction also play a significant role in solid state physics. The importance of quantum physics also reveals itself in solid state, and said properties will be discussed. This informative exploration of the mathematics involved in the foundation of advanced technology illustrates the symbiosis that exists between mathematics and physics thus bringing amazing advancements to our technological world that we live in today.

SLIME MOLDS AND PUBLIC TRANSIT

Stephen Kissler
University of Colorado, Boulder

Researchers Atsushi Tero et al. have published results of a mathematical model that mimics the network-forming behavior of the slime mold *Physarum polycephalum.* The mold's behavior is of great interest due to its ability to find the shortest path between two points in a labyrinth and, under the right conditions, to even model Japan's public transit system. These surprising traits result from the mold's ability to spontaneously form networks of high spatial efficiency and resilience in the event of a rupture. These networks may be pertinent to a range of fields beyond biology and engineering, including medicine and computer science. In this presentation, I will describe our work in developing of a discrete version of the Tero model and discuss future extensions of the model. The research is funded through the National Science Foundation's *Mentoring through Critical Transition Points* program.

ESTIMATION OF DISTRIBUTION OVERLAP FOR URN MODELS

Jerrad Hampton
University of Colorado, Bouder

A classical problem in statistics relates to estimating the probability covered by the outcomes observed in k draws from a given though unknown discrete distribution, the so-called coverage probability of the sample. This problem has been addressed, under various constraints and assumptions by Robbins, Starr, and Good-Turing, among many others. Potential applications of this include estimating the probability of discovering a new gene in large datasets of potential genes, as well as analyzing ancient coin hoards.

We consider an extension of the above problem to two discrete distributions. Namely, we are interested in estimating the probability that an outcome from one distribution is observed in k draws from the other. We address this problem modeling each distribution as an urn with colored balls occurring in unknown proportions. We assume no pre-existing information about the colors in each urn, and the only data available from each urn are samples with replacement of possibly different sizes. Our goal is to estimate m(k), the probability that a draw from Urn 1 is seen in k draws from Urn 2. We construct an unbiased estimator for m(k) by extending classical statistics from the one-urn problem. Expanding on results from U-statistics, we show this estimator is the UMVUE and address its asymptotic normality.

This research has been motivated by the problem of estimating the probability that a draw from one microbial environment is found in k draws from another, which provides an asymmetric measure of similarity between such environments. We demonstrate and interpret our estimates on data generated from ecologically relevant probability distributions. Furthermore, we use these estimates to decide which environment is most beneficial to further sample, and compare this decision to the decision implied from the corresponding one-urn problems.

CHARACTERIZATION OF 2-TREE UNIT PROBE INTERVAL GRAPHS

Shilpa Dasgupta
University of Colorado, Denver

In 1959 Benzer invented Interval Graphs to aid his work on DNA. Lots of studies have been done on these so far because of the variety of interesting structures they exhibits which could be used as useful tools in various other applications like physical mapping of DNA. In this paper we worked on a special type of interval graph called the unit probe interval graph. A probe interval graph (PIG) is a graph with vertex partition V = (P,N) and an interval assigned to each vertex so that vertices are adjacent if and only if their corresponding intervals overlap and at least one belongs to P. A unit probe interval graph (UPIG) is a PIG in which all intervals in some representation can be made to have same length. We took a special set of graphs called 2-trees and looked at the constraints that prohibit them from being a unit probe interval graph. Thus eventually deriving a complete characterization of 2-tree unit probe interval graphs.

ADAPTIVE COUPLING IN KURAMOTO NETWORKS OF OSCILLATORS

Kyla Maletsky
University of Colorado, Boulder

Many systems, such as groups of fireflies and sets of metronomes, can be modeled using coupled oscillators. The Kuramoto model is perhaps the most widely studied model for describing the synchronization of large systems of interacting oscillators. Due to the broad applicability of this model, new phenomena are continuously being observed for variations of this model, such as time delays, adaptive natural frequencies, and local coupling. In particular, recent research has explored networks of Kuramoto oscillators with adaptive coupling strengths. Although this research has uncovered interesting dynamics such as the formation and multi-stability of synchronized clusters, these results are restricted by the particular choice of coupling evolution. We propose an adaptive model that promotes local interactions by changing the coupling between a pair of oscillators at a rate that is dependent on their phase difference. In contrast to previous work, we control the effective phase difference ϵ below which oscillators will affect each other to address applications in neuroscience and animal mobility. We illustrate the mechanism of synchronization in our model by considering first the simple case of two oscillators. In numerical studies with a large number of oscillators, several synchronized clusters are observed, the number of which depends on the initial conditions and ϵ (as ϵ increases, the number of clusters decreases). Other aspects of the dynamics are also discussed.

AFTERNOON SESSION I

G-LATTICES OF AN UNROOTED PERFECT PHYLOGENY

Monica Grigg
Colorado School of Mines

In this talk we look at the Pure Parsimony problem and the Perfect Phylogeny Haplotyping problem. From the Pure Parsimony problem we consider structures of genotypes called g-lattices. These structures either provide solutions or give bounds to the pure parsimony problem. In particular, we investigate which of these structures supports an unrooted perfect phylogeny, a condition that gives a solution additional biological interpretation. By understanding which g-lattices support an unrooted perfect phylogeny, we connect two of the standard biological inference rules used to recreate how genetic diversity propagates across generations.

PREDICTING CRITICALITY AND DYNAMIC RANGE IN EXCITABLE COMPLEX NETWORKS: EFFECTS OF TOPOLOGY

Daniel Larremore
University of Colorado, Boulder

We study the effect of network structure on the dynamical response of networks of coupled discrete-state excitable elements which are stochastically stimulated by an external source. Such systems have been used as toy models for the dynamics of some human sensory neuronal networks and neuron cultures. The collective dynamics of such systems depends on the topology of the connections in the network. Here we develop a general theoretical approach to study the effects of network topology on dynamic range, which quantifies the range of stimulus intensities resulting in distinguishable network responses. We find that the largest eigenvalue of the weighted network adjacency matrix governs the network dynamic range. Specifically, a largest eigenvalue equal to one corresponds to a critical regime with maximum dynamic range. This result appears to hold for random, all-to-all, and scale free topologies, and is robust to the inclusion of time delays and refractory states. We gain deeper insight on the effects of network topology using a nonlinear analysis in terms of additional spectral properties of the adjacency matrix. We find that homogeneous networks can reach a higher dynamic range than those with heterogeneous topology. Our analysis, confirmed by numerical simulations, generalizes previous studies in terms of the largest eigenvalue of the adjacency matrix. These results provide a better understanding of the sources of enhanced dynamic range of neuronal networks and may be applicable to the study of other systems that can be modeled as a network of coupled excitable elements (e.g., epidemic propagation).

CLASSIFICATION OF THE KP EQUATION AND THE CYCLIC SHIFT

Tommy McDowell
University of Colorado, Colorado Springs

The Kadomtsev-Petviashvili(KP) equation is a partial differential equation in (x,y,t) that can be used to model shallow water wave mechanics. There exists an exact family of solutions. The construction of this solution will be established. I will also describe the classification and characteristics of the solution. Lastly, I will be introducing something known as the Cyclic Shift, which can be used to find more solutions to the KP equation, a non-trivial task.

PANCYCLICITY OF 4-CONNECTED, CLAW-FREE P_{10}-FREE GRAPHS

Timothy Morris
University of Colorado, Denver

A graph G is said to be pancyclic if G contains cycles of all lengths from 3 to $|V(G)|$. We show that if G is 4-connected, claw-free, and P_{10}-free, then G is either pancyclic or it is the line graph of the Petersen graph. This implies that every 4-connected, claw-free P_9-free graph is pancyclic, which is best possible and extends a result of Gould, Łuczak, and Pfender [R. Gould, T. Łuczak, and F. Pfender, Pancyclicity in 3-connected graphs: Pairs of forbidden subgraphs, *J. Graph Theory* **47** (2004), 183-202].

TOTAL DIFFERENTIALS, THERMODYNAMICS, AND POROUS MEDIA

Eric Sullivan
University of Colorado, Denver

When modeling fluid and gas flow through porous media the number of independent variables needed to accurately capture all of the physical processes can be overwhelmingly large. It therefore becomes necessary to make many simplifying assumptions in order to numerically or analytically solve these models. This is especially true in models that capture flow in unsaturated porous material with a swelling solid phase. A technique from physical chemistry which uses total differentials to switch between thermodynamic quantities will be presented. This technique will be used to demonstrate how these large complex models can be simplified by using an appropriate change of variables. Advantages and disadvantages of using this technique will be discussed.

THEORY AND SIMULATION OF CRITICAL DYNAMICS IN GINZBURG LANDAU MODELS

Ty Thompson
Colorado School of Mines

The Ginzburg Landau (GL) model of superconductivity is one of the most celebrated examples of the phenomenological modeling of theoretically complex physical systems. Since its initial inception, its predictions of vortex states in superconductors have been theoretically justified, numerically verified, and physically observed in many contexts. In this research, we focus on an instance of the time dependent GL model (TDGL) for thin superconductors having cylindrical symmetry about the axis of a constant magnetic field. Several fully discrete spectral Galerkin numerical simulation methods that utilize the underlying domain symmetry are developed, implemented, and evaluated. With these methods at hand, we examine some critical limitations of the model, and by relying on intuition demonstrate an approach for simulating vortex nucleation and motion associated with spatial symmetry loss. We use a previously introduced Langevin extension of the TDGL equation to more completely predict and quantify such transitions, and seek to numerically simulate the resulting stochastic partial differential equation (SPDE) with more advanced spectral methods based upon generalized spectral decompositions (GSD) in high dimension.

AFTERNOON SESSION II

Modeling Contest, Problem C

ANALYZING THE ECONOMIC AND ENVIRONMENTAL IMPACT OF WIDESPREAD ELECTRIC VEHICLE USE

Prince Agyeman, Monica Blomker, Priscus Kahangwa
University of Colorado, Denver

In understanding the current constraints and benefits of the electric vehicle industry, it was our goal to consider the practicality of widespread electric vehicle use. Through analyzing the cost of owning and maintaining an electric vehicle (EV) as opposed to an internal combustion engine vehicle (ICEV), as well as the yearly CO_2 emissions given off by each type of car, both in driving and in fuel production, we designed various models to estimate the overall costliness of electric versus internal combustion vehicles.

In our first model, we created a basic equation to determine the expense incurred by an EV versus ICEV owner; accounting for the initial price, yearly maintenance cost, and cost of fueling four compact cars, two EVs and two ICEVs. Our second and third models accounted for the environmental impact of each type of car over the span of seven years. We considered the amount of CO_2 emitted by driving as well as producing fueling agents for each vehicle. We targeted CO_2 emissions to better analyze the carbon footprint left by each vehicle.

Ultimately, we complied our results to determine which distribution of drivers (between EV and ICEV) would minimize overall monetary cost and environmental impact. Our findings suggest that an optimal level of 40% EV and 60% ICEV drivers would minimize the overall cost to producers, consumers, and the environment, therefore creating an ideal scenario for overall efficiency and practicality. Our results yielded 3.5×10^{10} gallons of gasoline saved per year, as well as a reduction in CO_2 emissions of 4.61×10^{11} kg annually.

Modeling Contest, Problem A

AIR WITH A TWIST: FINDING THE OPTIMAL SNOWBOARD HALF-PIPE

Marshall Carpenter, Keegan Boyle, Vicky Li
University of Colorado, Boulder

We develop a model to determine the snowboard half-pipe shape that maximizes vertical air and twist achieved while retaining practicality. Our model is based on the key assumptions that the snowboarder/board is equivalent to their center of mass, optimizing 2D cross-section shape optimizes half-pipe shape, optimizing exit velocity optimizes air, optimizing angular acceleration and air time optimizes twist, the half-pipe is symmetric and smooth and has vertical ends, the snowboarder gains velocity solely through pumping, the snowboarder has unlimited skill, and the only forces on the snowboarder are gravity and friction (from the normal and centripetal force). We derive a 2D cross-section model by combining the Force, Angular Momentum, and Energy models. The result is a powerful, general model which gives the snowboarders velocity with respect to angle of curvature not time, approximates discrete pumping, and accounts for friction and mountain slope. We then derive a 3D half-pipe model using a moving frame. Numerical implementation of the 2D model inputs cross-section and pumping shapes, along with parameters for friction, initial velocity, and velocity gain over the flat, and calculates velocity gain for various depths and widths. Results match reality and indicate that shape is optimized by the ellipse over the parabola, smallest dimensions, and most importantly practicality. Areas for improvement are: consideration of practical issues such as snowboarder strength and vertical end steepness, optimization of the flat and twist using the 3D model, and validation with real data.

MODELING ANT COLONY BEHAVIOR

Justin Baacke, Maxwell Bohning, Andrew Erickson, David Kozak, Patrick O'Brien
University of Colorado, Boulder

Research on colonies of the red harvester ant, Pogonomyrmex barbatus, has shown that the colony behaves as a complex system with each ant working towards the benefit of the colony without input or direction from any central source. There has been extensive field research to map out the basis behaviors of the individual ants and to assess the factors that influence how the colony operates from day to day. We have attempted to compile this research, and use it as a guideline for creating a program that simulates a day in the life of a colony. In the model, as in life, every ant reacts instinctively to its immediate surroundings, according to its basis behaviors, with no direction from the nest or the queen. That is, each ant knows only what it has experienced for itself and usually acts on that without regard to the colony's other current activities or needs. Despite the myopic attitude of its inhabitants, the colony is able to grow and prosper as long as there is food remaining. This agent based model of ant colonies has potential to reach across disciplines into more practical applications such as the stock market, the world economy, and the human nervous system, all of which are also complex systems.

POSITIVE EXTENSIONS OF MATRICES INDEXED BY A HOMOGENEOUS TREE

Jeremy Garcia
University of Northern Colorado

Let T be a homogeneous tree of order q – that is, an acyclic, undirected, connected graph such that every node belongs to exactly $q+1$ edges. Let T_n be a maximal subgraph of T with the property that the distance between any two nodes of T_n does not exceed n, and let $A = [a_{t,s}]_{t,s \in T_n}$ be a square positive definite matrix indexed by the nodes of T_n. The matrix A is said to be *isotropic* if $a_{t,s}$ depends only on the distance between the nodes t and s (in the case $q = 1$ this means that A is a real symmetric Toeplitz matrix). The positive extension problem for the matrix A consists in finding all such isotropic positive definite matrices B indexed by the nodes of $T_{n+1} \supset T_n$ that the diagonal block of B corresponding to T_n coincides with A.

This talk shall discuss a recursive solution of the positive extension problem formulated above. The talk is based on a joint work with D. Volok.

SYNCHRONIZATION IN VAN DER POL OSCILLATORS

Scott Pawlowski
University of Colorado, Boulder

The Van der Pol oscillator is studied in detail. We begin by exploring the single oscillator and it's characteristics such as limit cycle, frequency, and mode locking. We also explore how the single oscillator reacts to a forcing function. We then examine two oscillators coupled with a common factor where we study the same characteristics as with the single oscillator. The coupled system is more interesting dynamically so we apply advanced methods to the try to gain more qualitative and quantitative insight into the Van der Pol system. Specifically we utilize a method of partial fraction expansion in order to determine the mode locking regions of the system. We examine the behavior in various phase spaces in order to find trends in the system.

PERMEABILITY OF FLUID FLOW THROUGH CILIA

Kannanut Chamsri
University of Colorado, Denver

This work concentrates on finding the permeability for viscous flow though a structure of periodic cylinders as a function of cylinder density and cylinder angle. We use a full three-dimensional model of incompressible viscous fluid in combination with the Buckingham Pi Theorem to determine the relationships. Numerical results are obtained using Mixed Finite Element Method. The results are compared with Rodrigo P.A. Rocha and Manuel E. Cruz with a good agreement. Applications include modeling the flow of mucus in lung tissue.