Fifth Annual Front Range Applied Mathematics Student Conference

March 14, 2009

Breakfast and Registration: 8:30 - 8:55

Morning Session I - Room NC1605
9:00 - 11:00

9:00 - 9:20	Jeffrey Larson *University of Colorado, Denver*	An Implementation of Scatter Search to Classify Medical Images
9:25 - 9:45	Donald McCuan *University of Colorado, Denver*	Eigensolvers for Analysis of Microarray Gene Expression Data
9:50 - 10:10	Daniel Kaslovsky *University of Colorado, Boulder*	Performance of Empirical Mode Decomposition on Noisy Data
10:15 - 10:35	Tim Lewkow *University of Colorado, Colo. Springs*	Line-Soliton Solutions of the KP Equation
10:40 - 11:00	Nathan Halko *University of Colorado, Boulder*	A Randomized Algorithm for the Approximation of Low Rank Matrices

Morning Session II - Room NC1603
9:00 - 11:00

9:00 - 9:20	Matthew Nabity *University of Colorado, Denver*	On the use of pivoted Cholesky in the CholeskyQR algorithm
9:25 - 9:45	Christian Ketelsen *University of Colorado, Boulder*	A Transformation of the 2D Dirac Equation of Quantum Electrodynamics
9:50 - 10:10	Keith Wojciechowski *University of Colorado, Denver*	A High-Order Numerical Solution to a Partial Integrodifferential Equation Modeling a Swelling Porous Material
10:15 - 10:35	Anil Damle, Geoffrey Peterson *University of Colorado, Boulder*	Finding the Eigenstructure of Isoceles Triangles Using McCartin's Method
10:40 - 11:00	Elizabeth Untiedt *University of Colorado, Denver*	A Survey of Applications for the Mathematics of Fuzzy Sets

Break: 11:00 - 11:15

Plenary Address: 11:15 - 12:15, Plaza M205

Mark Newman, *University of Michigan*
Epidemics, Erdos numbers, and the Internet: The structure and function of complex networks

Lunch: 12:15 - 1:00

Afternoon Session I - Room NC1605
1:00 - 3:00

1:00 - 1:20	Michael Presho *University of Wyoming*	The Multiscale Finite Element Method
1:25 - 1:45	Kye Taylor *University of Colorado, Boulder*	Automatic Detection and Identification of Seismic Waves
1:50 - 2:10	Christian Hampson *Colorado State University*	Characteristics of Certain Families of Random Graphs
2:15 - 2:35	Lei Tang *University of Colorado, Boulder*	Parallelization of Efficiency-Based Adaptive Local Refinement for FOSLS-AMG
2:40 - 3:00	Douglas Baldwin *University of Colorado, Boulder*	Soliton Generation and Multiple Phases in Dispersive Shock and Rarefaction Wave Interaction

Afternoon Session II - Room NC1603
1:00 - 3:25

1:00 - 1:20	Yang Zou *Colorado State University*	Evolution of Quantitative Traits with Immigration
1:25 - 1:45	Minho Park *University of Colorado, Boulder*	A New Least Squares Based AMG
1:50 - 2:10	Doug Lipinski *University of Colorado, Boulder*	Lagrangian Coherent Structures: An Application to Jellyfish Feeding
2:15 - 2:35	Jennifer Maple *Colorado State University*	Characterization of Spatio-Temporal Complexity in Ginzburg-Landau Equations for Anisotropic System
2:40 - 3:00	Christopher Harder *University of Colorado, Denver*	A PEGM Method for the Darcy Problem
3:05 - 3:25	Ryan Kennedy *University of Colorado, Boulder*	A Framework for Pattern Recognition in Molecular Biology Data

Afternoon Session III - Room NC1806
1:00 - 3:25

1:00 - 1:20	Yongli Chen, Tim Lewkow *University of Colorado, Colo. Springs*	*Modeling Contest, Problem B* Estimating the Impact of Wireless Communications Growth on Energy Consumption and the Environment
1:25 - 1:45	Lee Rosenberg, Michelle Rendon, Manuchehr Aminian *University of Colorado, Denver*	*Modeling Contest, Problem B* Modeling Energy Consumption of the Current Telecommunications Structure
1:50 - 2:10	Anil Damle, Anna Lieb, Geoffrey Peterson *University of Colorado, Boulder*	*Modeling Contest, Problem A* Pseudo-Finite Jackson Networks and Simulation: A Roundabout Approach to Traffic Control
2:15 - 2:35	Ruth Martin *University of Colorado, Boulder*	An Investigation of the Linear Shallow Water Wave Equations in One-Dimension
2:40 - 3:00	Joseph Adams, Ryan Schilt *University of Colorado, Boulder*	Experiments on Mixing: Aref's Blinking Vortex
3:05 - 3:25	Tyler Takeshita *University of Northern Colorado*	A Systematic Circuit Approach to Modeling Neurons with an Ion Pump

Plenary Speaker

11:15 - 12:15, PLAZA M205

Epidemics, Erdos numbers, and the Internet:
The structure and function of complex networks

Mark Newman, University of Michigan

There are networks in almost every part of our lives. Some of them are familiar and obvious: the Internet, the power grid, the road network. Others are less obvious but just as important: the patterns of friendships or acquaintances between people form a social network; the species in an ecosystem join together to form a food web; the workings of the body's cells are dictated by a metabolic network of chemical reactions. As large-scale data on these networks and others have become available in the last few years, a new science of networks has grown up, drawing on ideas from math, engineering, biology, physics and other fields to shed light on systems ranging from bacteria to the whole of human society. This lecture will look at some new disoveries regarding networks, how these discoveries were made, and what they can tell us about the way the world works.

Mark Newman is the Paul Dirac Collegiate Professor of Physics at the University of Michigan. He is affiliated with the department of physics and the Center for the Study of Complex Systems. His research focuses on the structure and function of networks. These include scientific coauthorship networks, citation networks, email networks, friendship networks, epidemiological contact networks, and animal social networks. He has investigated fundamental network properties such as degree distributions, centrality measures, assortative mixing, vertex similarity, and community structure. He has also made analytic or computer models of disease propagation, friendship formation, the spread of computer viruses, the Internet, and network navigation algorithms.

MORNING SESSION I

AN IMPLEMENTATION OF SCATTER SEARCH TO CLASSIFY MEDICAL IMAGES

Jeffrey Larson
University of Colorado, Denver

While attempting to train neural networks to classify magnetic resonance images (MRIs) of the brain as normal or abnormal, back-propagation frequently performs poorly due to familiar problems with gradient descent and local optima. Here we propose an implementation of the scatter search metaheuristic to circumvent the problems of back-propagation to distinguish between human MRIs with multiple sclerosis lesions and normal MRIs. We increase by at least 10% the classification rate using scatter search versus back-propagation.

EIGENSOLVERS FOR ANALYSIS OF MICROARRAY GENE EXPRESSION DATA

Donald McCuan
University of Colorado, Denver

DNA microarrays provide gene expression data for use in research on disease and metabolic processes. This research focuses on high level analysis of micro array data to identify genes with similar expression profiles. We use the method of spectral clustering and apply eigensolvers to identify clusters of genes.We will give an overview of gene expression and DNA microarrays, discuss clustering of data with emphasis on spectral analysis, graph Laplacians and use of eigensolvers (Blopex) in the analysis.

PERFORMANCE OF EMPIRICAL MODE DECOMPOSITION ON NOISY DATA

Daniel Kaslovsky
University of Colorado, Boulder

While most data collected in scientific applications are nonstationary, the most well known and widely used methods for analysis assume stationary data. Empirical Mode Decomposition (EMD) is an algorithm that adaptively decomposes nonstationary data into basis elements called Intrinsic Mode Functions (IMFs). The key feature of this decomposition is each IMF holds properties that allow for a well defined Hilbert Transform and thus gives rise to a physically meaningful instantaneous frequency.

We investigate the performance of EMD on a constructed class of signals. We examine the ability of EMD to produce a meaningful instantaneous frequency by testing the limits of the algorithm in the presence of noise. In doing so we explore the ability of EMD to identifying and remove noise from a given signal. The denoising capabilities are tested against two standard denoising techniques.

LINE-SOLITON SOLUTIONS OF THE KP EQUATION

Tim Lewkow
University of Colorado, Colo. Springs

This research project involves the study of a weakly two dimensional version of the Kortweg-deVries equation, known as the Kadomtsev-Petviashvili (KP) equation. The KP equation models nonlinear wave inter- actions in shallow water. Recent works have also considered the KP model to describe the mechanisms causing extremely high elevation waves on the ocean surface. There exist many different classes of solutions for the KP equation, however the focus of this research work is on the so called line-soliton solutions. This paper describes some specic details of one particular line-soliton solution known as O-type 2-soliton. Along with the structure of this solution, and some details involving the amplitude at the interaction point, a combinatorial problem is also introduced.

A RANDOMIZED ALGORITHM FOR THE APPROXIMATION OF LOW RANK MATRICES

Nathan Halko
University of Colorado, Boulder

In many cases it is desirable to have a low rank factorization of a matrix such as a Singular Value Decomposition (SVD). As the first step of obtaining the SVD, an orthonormal basis of the matrix's column span is computed via our randomized algorithm. Then using the basis to reduce the size of the matrix, the SVD can be computed for much less cost than working directly with the original matrix. We consider a class of matrices who's singular values decay exponentially. In this case, the rank of the matrix is much less than the dimension of the span which gives us a good setting to use randomized sampling. However, we also have a mechanism to deal with slower decay. A discussion of the algorithm will detail our adaptive rank determination and error checking procedures. Numerical examples will be presented that illustrate the robustness and precision of the resulting approximations.

MORNING SESSION II

ON THE USE OF PIVOTED CHOLESKY IN THE CHOLESKY-QR ALGORITHM

Matthew Nabity
University of Colorado, Denver

Given a nonsingular m-by-n matrix A ($m \geq n$), the CholeskyQR algorithm computes the QR factorization of the matrix A. In practice, it is by all means the fastest known algorithm for the QR factorization of a tall and skinny matrix and it indeed achieves (in the big $\mathcal{O}$ sense) lower bounds for latency, bandwidth and number of operations. It is also an extremely simple algorithm to formulate. Using Matlab notations, the algorithm reads:

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R = A'*A; R = chol(R); Q = A / R;
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Unfortunately, this algorithm is unstable and, for ill-conditioned matrix A, the resulting vectors Q do not form an orthonormal basis. ($\|I - Q^TQ\| \gg \varepsilon$.) And indeed for $\kappa(A) > 1/\varepsilon$, it is likely that the algorithm breaks down in the Cholesky factorization phase. (In exact arithmetic, A nonsingular guarantees A^TA symmetric positive definite. In floating point arithmetic, A^TA is symmetric (by construction) but can be indefinite. Therefore Cholesky breaks down.)

In order for the algorithm to succeed in the Cholesky factorization phase, several variants have been proposed. All of the variants we have found in the literature need the eigenvalue decomposition of the symmetric matrix A^TA. If we assume $m \gg n$, then the eigenvalue decomposition of the symmetric matrix A^TA is an $\mathcal{O}(n^3)$ steps which is negligible with respect to the two other operations ($\mathcal{O}(mn^2)$). However, in actuality, this step can significantly increase the overall time to solution compared to a Cholesky factorization. We propose to perform a pivoted Cholesky factorization of the matrix A^TA to cure the problem of the breakdown in Cholesky.

With this modification, CholeskyQR runs without breakdown; however the resulting matrix Q can still be far from orthogonal. In that case, a reorthogonalization step is needed. We explain how to perform reorthogonalization in the context of CholeskyQR.

A TRANSFORMATION OF THE 2D DIRAC EQUATION OF QUANTUM ELECTRODYNAMICS

Christian Ketelsen
University of Colorado, Boulder

The Schwinger Model of quantum electrodynamics (QED) describes the interaction between electrons and photons. Large scale numerical simulations of the theory require repeated solution of the two-dimensional Dirac equation, a system of two first order partial differential equations coupled to a background U(1) gauge field. Traditional discretizations of this system are sparse and highly structured, but contain random complex entries introduced by the background field. For even mildly disordered gauge fields the near kernel components of the system are highly oscillatory, rendering standard multilevel methods ineffective.

We consider and alternate formulation of the governing equations obtained by a transformation of the continuum operator which essentially eliminates the gauge field. The resulting formulation resembles uncoupled diffusion-like problems with variable diffusion coefficients. We discuss the implications of the transformed formulation in terms of the physical theory as well as its amenability to discretization and solution by least-squares finite elements and adaptive multilevel methods, respectively.

A HIGH-ORDER NUMERICAL SOLUTION TO A PARTIAL INTEGRODIFFERENTIAL EQUATION MODELING A SWELLING POROUS MATERIAL

Keith Wojciechowski
University of Colorado, Denver

We demonstrate the results of a discrete scheme used to solve a nonlinear model of a swelling porous material. The model is cast as a Volterra partial integrodifferential equation (VPIDE) of the second kind where the dependent variable is the liquid phase volume fraction. Pseudospectral differentiation matrices are constructed to compute the spatial derivatives in polar coordinates and a Volterra Runge-Kutta scheme is used to solve the VPIDE at each time step. Some comparisons, in terms of accuracy and stability, will be made between different constructions for the polar Laplacian. Results will be shown and physical interpretations given for the time series plots of the volume fraction as well as the moisture content and viscoelastic stresses.

FINDING THE EIGENSTRUCTURE OF ISOSCELES TRIANGLES USING MCCARTIN'S METHOD

Anil Damle, Geoffrey Peterson
University of Colorado, Boulder

In this presentation, we will discuss a method to obtain solutions to Laplace's Equation within a triangular region under Dirichlet conditions. Originally presented by Brian J. McCartin in his paper "Eigenstructure of the Equilateral Triangle, Part I: The Dirichlet Problem," (*Siam Review*, Vol. 45, No. 2, pp. 267-287), this method relies on Lame's Fundamental Theorem to construct solutions that are either symmetric or antisymmetric with respect to an altitude of the triangle.

We first discuss the coordinate system that is used to easily parameterize the isosceles triangle and the obtained solution. The Cartesian coordinates of points inside the triangle must be transformed into a triangular coordinate system which measures the distance from the intersection point of the altitudes to the projection of the point onto the altitude. With this new coordinate system, we also must redefine the Laplace operator in order to properly use separation of variables.

We then construct a form of the solution that is symmetric about the bisecting altitude of the isosceles triangle and zero along the edges using sine and cosine functions. We then analyze these solutions to find six equations for three eigenvalues, only to find that the only case in which consistent solutions exist for these equations is the equilateral triangle. Although we can choose certain eigenvalues that satisfy the Dirichlet conditions, we show graphically that they may not also satisfy Laplace's Equation by analyzing certain extreme conditions.

We conclude our discussion by looking at conditions under which complete solutions could exist, although we may not be able to find them using McCartin's method. These conditions include the ability to tile the plane with the triangle solely through rectangular reflections, the location of nodal lines in extensions of the solution, and whether the solutions are symmetric or anti-symmetric. From this discussion, we present triangles where we believe that solutions should not exist and triangles where certain types of solutions may exist, such as the isosceles right triangle.

A SURVEY OF APPLICATIONS FOR THE MATHEMATICS OF FUZZY SETS

Elizabeth Untiedt
University of Colorado, Denver

Since their introduction by Lofti Zadeh in 1960, fuzzy sets have found widespread application in engineering and computing. Some branches of fuzzy set theory have flourished in implementation, and others, while theoretically well-developed, have enjoyed little practical application.

This talk will provide an outline of fuzzy set theory, with a brief overview of successful, unsuccessful, and potential application of each branch.

Special attention will be paid to fuzzy optimization theory, as well as to the author's own work in radiation therapy planning, heating oil distribution, and structured financial products.

AFTERNOON SESSION I

THE MULTISCALE FINITE ELEMENT METHOD

Michael Presho
University of Wyoming

The study of reservoir fluid flow is a broad research topic with plenty of room for advancement. In many cases existing data is sparse, and as a result, a number of reservoir uncertainties and scales can arise. Mathematically speaking, the model equations of interest often include multi-scale variables. For the presentation I will consider two-dimensional pressure and concentration equations which describe flow in a porous medium. The heterogeneous porous medium is described through small-scale assumptions that are made on the initial permeability data, and the equations are solved using the multiscale finite element method (MsFEM). The goal of MsFEM is to incorporate the small-scale data into basis functions which can be used to treat the large scale problem. Through using MsFEM, we effectively solve on a coarse mesh while still maintaining the effects of the initial fine scale data.

AUTOMATIC DETECTION AND IDENTIFICATION OF SEISMIC WAVES

Kye Taylor
University of Colorado, Boulder

We address the problem of automatically detecting and identifying regional seismic phases (Pg, Pn, Lg, Sn) from a single-component, or a three-component, seismogram. Operating under the assumption that seismic signals organize themselves around a smooth manifold, we use discrete approximations to the manifold's vibratory modes to obtain a sparse representation for each of the seismic signals. These new representations are not only faithful to the manifold structure influencing the data, but provide a way to analyze the data that is not cursed by the large number of samples measured in the time-domain. Once the new representations are determined, we train a classifier to first detect whether or not a seismic phase is present, and if so, assign a specific phase to the unlabeled data. Our algorithm is compared against industry standards.

CHARACTERISTICS OF CERTAIN FAMILIES OF RANDOM GRAPHS

Christian Hampson
Colorado State University

Applications of random graphs include prediction of disease and habitat invasion processes. Families of edge-product random graphs can be characterized by their probability matrix. The distributions of some attributes of these graphs can then be determined directly from this probability matrix. A theorem by Hermann Weyl is then used to characterize some elements of the spectra of these families of edge-product random graphs.

PARALLELIZATION OF EFFICIENCY-BASED ADAPTIVE LOCAL REFINEMENT FOR FOSLS-AMG

Lei Tang
University of Colorado, Boulder

A new adaptive local refinement strategy (ACE), in which the refinement decisions are based on optimizing the computational efficiency, has recently been developed. The first-order system least-squares (FOSLS) finite element methods, together with algebraic multigrid methods (AMG), complement this idea, resulting in a powerful tool for numerical solutions of many PDEs. However, in parallel environment, the global sorting of element local error indicators used by the original approach results in undesirable communication between processors. A strategy circumventing the global sorting is proposed. The test results show that the strategy has the potential of greatly reducing communication, while producing a sequence of refined grids with essentially the same quality as before.

SOLITON GENERATION AND MULTIPLE PHASES IN DISPERSIVE SHOCK AND RAREFACTION WAVE INTERACTION

Douglas Baldwin
University of Colorado, Boulder

Interactions of dispersive shock (DSWs) and rarefaction waves (RWs) associated with the Korteweg-de Vries equation are shown to exhibit multiphase dynamics and isolated solitons. There are six canonical cases: one is the interaction of two DSWs which exhibit a transient two-phase solution, but evolve to a single phase DSW for large time; two tend to a DSW with either a small amplitude wave train or a finite number of solitons, which can be determined analytically; two tend to a RW with either a small wave train or a finite number of solitons; finally, one tends to a pure RW.

AFTERNOON SESSION II

EVOLUTION OF QUANTITATIVE TRAITS WITH IMMIGRATION

Yang Zou
Colorado State Universtiy

Ecological and genetical changes occur simultaneously but on different time scales and evolution is usually much slower than ecological changes. The interactions between them lead to the fast-slow dynamical system. In this paper, a model describing evolution of the mean value of a quantitative trait with a direct migration scheme is proposed, and numerical methods are applied to test assumptions and the robustness of the approximate equation. Finally, a fast-slow dynamical system, which relates the immigration to the ecological subsystem is set up and analyzed numerically.

A NEW LEAST SQUARES BASED AMG

Minho Park
University of Colorado, Boulder

In this talk we compare Brandt's BAMG with indirect Bootstrap Algebraic Multigrid(iBAMG) that determines the interpolation weights by collapsing F-F connections in the operator equation. Solving the linear system using least squares based AMG requires set of vectors that are results of several fine level relaxation sweeps on homogeneous equation $Ae = 0$. Unlike Brandt's BAMG, new method approximate all F-F connections in least squares sense. The presented numerical experiments demonstrate that the method can achieve good convergence with less vectors and relaxation sweeps.

LAGRANGIAN COHERENT STRUCTURES: AN APPLICATION TO JELLYFISH FEEDING

Doug Lipinski
University of Colorado, Boulder

Lagrangian coherent structures (LCS) represent a relatively recent development in the application of certain dynamical systems theories to time dependent systems. Specifically, LCS define a time dependent analogue to the stable and unstable manifolds of homogeneous dynamical systems. In this talk, I will give a brief overview of LCS, including a rigorous definition and properties as outline by Shadden et. al in 2005 and follow this with an application of LCS to flow around a swimming jellyfish. We find that the jellyfish Aequorea victoria creates a flow which aids in feeding while swimming due to its "paddling" or "rowing" type of propulsion. We note that only certain jellyfish feed while swimming and present a further example of a "jetting" type of jellyfish, Sarsia tubulosa, as a contrast.

CHARACTERIZATION OF SPATIO-TEMPORAL COMPLEXITY IN GINZBURG-LANDAU EQUATIONS FOR ANISOTROPIC SYSTEM

Jenifer Maple
Colorado State University

A theoretical and numerical study of a system of four globally coupled complex Ginzburg-Landau equations, derived from the weak electrolyte model of nematic electroconvection and describing the steady oblique Hopf normal travelling interaction in an anisotropic extended system is presented. The model was derived. The system is simulated numerically using a pseudo-spectral method to compute the solution to these equations and provide a visualization tool to study the bifurcations in the system and its spatio-temporal complex behavior.

A PEGM METHOD FOR THE DARCY PROBLEM

Christopher Harder
University of Colorado, Denver

It is well-known that the discrete pair of spaces used in solving the Darcy problem by finite elements must satisfy an inf-sup condition. This places fairly severe limitations on the combination of spaces that can be used in a method. In particular, the element using piecewise linear velocity and piecewise linear pressure (both discontinuous and continuous) interpolations lead to an ill-posed linear system of equations. We develop a method which allows for the use of this pair of spaces by using the Petrov-Galerkin Enrichment Method (PGEM) to achieve stabilizing terms arising from a multiscale decomposition of the solution. The underlying approach correlates the enriched method to some local stabilized projection methods making the latter consistent. Numerical results show optimal convergence in natural norms and validate theoretical results.

A FRAMEWORK FOR PATTERN RECOGNITION IN MOLECULAR BIOLOGY DATA

Ryan Kennedy
University of Colorado, Boulder

In many biological systems such as gene regulatory networks, RNA splicing, and RNA polyadenylation, the individual parts of the system can interact in complex, nonlinear ways. As large numbers of biological sequences are accumulated, it is necessary to find methods that can uncover these complex relationships. Toward this goal, we present a framework of machine-learning techniques that can be used to identify patterns in data taken from systems in molecular biology. These techniques are able to identify interactions relating to differences within individual sequences - including covariation between positions - as well as interactions between the sequences themselves. We demonstrate the use of these methods in looking for patterns related to the transcription factor p53, a protein known to have an effect on tumor suppression.

AFTERNOON SESSION III

Modeling Contest, Problem B

ESTIMATING THE IMPACT OF WIRELESS COMMUNICATIONS GROWTH ON ENERGY CONSUMPTION AND THE ENVIRONMENT

Yongli Chen, Tim Lewkow
University of Colorado, Colo. Springs

Over the past twenty years, the worldwide cell phone industry has grown at an exponential rate. In the United States alone, the number of cell phone users has grown from 340,000 in 1985 to over 260 million in 2008. The growing number of cell phone users significantly contributes to the overall energy requirement in the United States and abroad. Additionally, because of the low cost of developing wireless infrastructure compared to that of wide spread land line development, many developing countries are turning to wireless networks to meet their communications needs. The purpose of this project was to estimate the impact of wireless communications growth on energy consumption and the environment, compared to that of landline development, in order to determine an optimum method of implementing residential phone service in emerging markets.

Modeling Contest, Problem B

MODELING ENERGY CONSUMPTION OF THE CURRENT TELECOMMUNICATIONS STRUCTURE

Lee Rosenberg, Michelle Rendon, Manuchehr Aminian

University of Colorado, Denver

Amid the fears of global warming, ozone depletion, climate change, and global security, the general acceptance among the public is that something needs to change in our energy usage habits. Industry and low-efficiency automobiles are obvious culprits, but what other ways can we cut back on oil usage before it is too late? Some potential culprits worth examining are the many electronics we have come to take for granted, including cell phones. In the past decade, cell phones have evolved from a luxury to necessity. But what effect, if any, has the change had on our rate of oil consumption? We attempt to model the current telecommunications situation, and in the process, take a closer look at some seemingly innocent electrical devices, and ask a few What if's?

We used two models to gain a grasp of the current state of the telecommunications industry – an iterative C++ program and a Linear Programming (LP) model. The C++ program treated the situation on several different levels, including the household, city, region, and national levels. The program randomized individuals and households around given sample mean (from research) to reflect the probabilistic nature of the problem, and kept track of quantities such as each individual's cell phone's battery life, as well as when and how often a cell phone is charged.

The second model, the Linear Programming model, examined closer what constraints would prevent an optimal solution simply being an extreme situation with either 100% landlines or 100% cell phones. The two models' results closely mirrored each other, and both models agreed that neither extreme (that is, 100% cell or 100% landlines) was optimal, and that a relatively cellular-phone-heavy mixture was highly efficient.

Using the iterative C++ program, it was found that an optimal combination, both by energy efficiency and power-grid usage stability, was the use of 85.4% cellular phones and 14.6% landlines. When the simulation was run with these optimal parameters, a total energy savings over the Inevitable Energy Usage, or basically, the energy usage of a mediocre solution (neither efficient nor inefficient), was around 4.4TWhr in the simulated time period. This translates to a savings of 2,588,413.79 barrels of oil over the aforementioned mediocre solution in the same simulated period of time.

After looking at the large picture, we investigated the power draw of simple household electronic devices, even when they were seemingly off. Surprisingly, many drew significant wattage in idle and off states, and one even drew the same wattage regardless of whether it was on or off!

In doing this, we hoped to clarify some common perceptions and misperceptions that people have about their electronics, and to provide a broader picture to the concept of doing one's part to help the environment energy efficiency on the consumer end does not always translate to greater energy efficiency on the national or global level.

Modeling Contest, Problem A
PSEUDO-FINITE JACKSON NETWORKS AND SIMULATION: A ROUNDABOUT APPROACH TO TRAFFIC CONTROL

Anile Damle, Anna Lieb, Geoffrey Peterson
University of Colorado, Boulder

In response to the 2009 MCM modeling problem, we create and analyze an analytical model of traffic flow in roundabouts with the purpose of optimizing traffic flow through the use of traffic lights or yield signs. To accomplish this task, we use an analytic model to find an optimum, defined by time spent in the roundabout system, solution and then apply a computer simulation to corroborate these results.

We use the concept of Jackson networks to create an analytic model. A roundabout can be thought of as a network of queues, where the entry queues receive external arrivals, which move into the roundabout queue before exiting the system. To form this model, we must assume that an equilibrium state may exist and that arrival rates are constant. The Jackson network is useful because, if certain conditions are met, a closed form stationary distribution may be found. Furthermore, the parameters for this system may be obtained empirically: how often cars arrive at an entrance (external arrival rate), how quickly they may enter the roundabout (internal arrival rate), and how quickly they exit (departure rate). We account for the traffic control method by thinning the internal arrival process with a “signal” parameter that represents the fraction of time that a signal light is green.

One pitfall of this formulation is that restricting the capacity of the roundabout queue to a finite limit will destroy the useful analytic properties of the system. We utilize a “pseudo-finite” capacity formulation, where we allow the roundabout queue to receive a theoretically infinite number of cars, but optimize over the signal parameter to create a steady state in which a minimal number of waiting cars is overwhelmingly likely. Using lower bound calculations, we prove that the yield sign produces the optimal behavior for all sets of allowed parameters. The analytic solution, however, sacrifices important aspects of a real roundabout, such as time-dependent flow.

To test the theoretical conclusions, we develop a computer simulation which incorporates more parameters: roundabout radius, car length, car spacing, car velocity inside the roundabout, periodicity of traffic signaling, and time-dependent input flow rates. The simulation uses these parameters to stochastically model individual vehicles as they move through the system, resulting in more realistic output. In addition to comparing yield and traffic signal control, we also examined variable distributions of input rates, non-standard roundabout constructions, and the relationship between traffic flow volume, radius size, and average total time. Our simulation is, however, limited to a single-lane roundabout. This model is also compromised by the very stochasticity that enhances its realism. Since it is nondeterministic, there is a good deal of randomness which may mask the true behavior. Another drawback of the stochastic formulation is that, as we show, the computational cost of mathematical minimization is enormous. We use our background research to set up the model as an empirical experiment to verify the hypothesis that a yield sign is almost always the best form of flow control.

AN INVESTIGATION OF THE LINEAR SHALLOW WATER WAVE EQUATIONS IN ONE-DIMENSION

Ruth Martin
University of Colorado, Boulder

The devastating effects of the 2004 tsunami have motivated increased investigation into the modeling of water waves. To this end, the shallow water (SW) equations, which can describe the behavior of tsunami waves in the ocean, are investigated. The SW equations are partial differential equations that model the propagation of water waves when the horizontal length scale is much greater than the vertical length scale. These equations are derived from Euler’s equations in both Cartesian and polar coordinates. A linearization of the SW equations is considered. Further restrictions are imposed so that the problem is one-dimensional in Cartesian and axisymmetrical in polar coordinates. The method of characteristics is used in both cases to develop a numerical scheme for solving the equations. The flow has exact invariants, which are used to check the accuracy of the computations.

EXPERIMENTS ON MIXING: AREF’S BLINKING VORTEX

Joseph Adams, Ryan Schilt
University of Colorado, Boulder

The motion of a passive scalar in a two-dimensional, incompressible, steady flow is essentially a Hamiltonian system with a single degree of freedom, with the stream function playing the role of Hamiltonian. Interestingly, the introduction of even very simple time dependence may transform this integrable system into a chaotic one. The term ”chaotic advection” was introduced by Hassan Aref to describe the non-integrable flow which arises in time-dependent, but non-turbulent fluids. One idealized model is Aref’s blinking vortex, which consists of two stirrers in a larger cylinder filled with viscous fluid. Each stirrer spins for a certain period of time, then stops while the other cylinder runs for a certain period of time. We seek to create and compare analytic, numerical, and experimental realizations of chaotic advection.

A SYSTEMATIC CIRCUIT APPROACH TO MODELING NEURONS WITH AN ION PUMP

Tyler Takeshita
University of Northern Colorado

Motivated by Professor Bo Deng's work, a systematic circuit approach to modeling neurons with an ion pump is presented. Like Dr. Deng, the voltage-gated current channels of a neuron are modeled as conductors, the diffusion-induced current channels are modeled as negative resistors, and the one-way ion pumps are modeled as one-way inductors. This model differs from the well-known Hodgkin-Huxley model because it splits the active and the passive branches of each ion species where as the HH approach combines the electromagnetic, diffusive, and pump channels of each ion into one conductance channel. Our model maintains several of the known properties of HH models along with being rich in many new dynamical structures including chaotic behavior.