Program for the 2007 SIAM Front Range Applied Mathematics Student Conference

March 3rd, 2007

Breakfast and Registration: 8:30 - 9:00

Morning Session I - Room 1603
9:00 - 10:35

9:00 - 9:20	Brandon Booth, Ben Safdi, and Kye Taylor *CU - Boulder*	Ising Model Applied to Congressional Redistricting - MCM 2007
9:25 - 9:45	Sanghui Lee, John Villavert and Nathan Schill *CU - Colorado Springs*	The Kidney Exchange Problem - MCM 2007
9:50 - 10:10	Lee Rosenberg, Derlin Campbell, and Lydie Van Holland *CU - Denver*	Airplane Boarding Model - MCM 2007
10:15 - 10:35	Jisun Lim *CU - Boulder*	Stability of Solutions to a Reaction Diffusion System Based Upon Chemical Reaction Kinetics

Morning Session II - Room 1605
9:00 - 10:35

9:00 - 9:20	Jutta Bikowski *Colorado State University*	Electrical Impedance Tomography and the Pioneering Work of A. Calderon
9:25 - 9:45	Ethan Murphy *Colorado State University*	Reconstructions of Conductive and Insulating Targets Using the D-Bar Method on an Elliptic Domain
9:50 - 10:10	Mike Watson *CU - Boulder*	Fluid Dynamics in Cylindrical Geometries: Simplifying Navier-Stokes for Geophysically Relevant Problems
10:15 - 10:35	Benjamin Safdi *CU - Boulder*	Finite-Time Singularities in Coupled Nonlinear Schrödinger Equations with 4-wave Mixing

Break: 10:35 - 10:45

Plenary Address, Leslie Greengard: 10:45 - 11:45, North Classroom 1607

The Nonuniform FFT and Magnetic Resonance Image Reconstruction

Lunch: 12:00 - 1:00

Afternoon Session I - Room 1603
1:00 - 4:00

1:00 - 1:20	Lauren Anderson *CU - Boulder*	Isolation and Implementation of the Dynamical Core from the German Weather Service's Numerical Weather Prediction Model
1:25 - 1:45	Josh Nolting *CU - Boulder*	Local Adaptive Refinement (LAR)FOSLS
1:50 - 2:10	Kye Taylor *CU - Boulder*	Local Geometric Projection for Denoising M-Valued Data
2:10 - 2:20	**Short Break**	
2:25 - 2:45	Angela Harris *CU - Denver*	H-avoiding Hamiltonian Cycles
2:50 - 3:10	Geoffrey Sanders *CU - Boulder*	Using a Generalized Eigensolver Based on Smoothed Aggregation (GES-SA) to Initialize Smoothed Aggregation Multigrid
3:15 - 3:35	Eugene Vecharynski *CU - Denver*	Solution of Large Sparse Underdetermined Linear Systems with Embedded Network Structure for a Non-Homogeneous Network Flow Programming Problem
3:40 - 4:00	Keith Wojciechowski *CU Denver*	An Introduction to Spectral Methods via the Numerical Solution of a Fluid Transport Equation

Afternoon Session II - Room 1605
1:00 - 4:00

1:00 - 1:20	Gisella Kagy *CU - Boulder*	The Impact of Conditional Cash Transfers on Prenatal Health: Is It Just a Supply Effect?
1:25 - 1:45	Robert Engle *CU - Colorado Springs*	Jukes-Cantor DNA Probability Model
1:50 - 2:10	Jen-Mei Chang *Colorado State University*	Illumination Face Spaces are Idiosyncratic
2:10 - 2:20	**Short Break**	
2:25 - 2:45	Minjeong Kim *CU - Denver*	Mathematical and Numerical Modeling of Wildfire

2:50 - 3:10	Bedrich Sousedik *CU - Denver*	A Note on the Patch Test for the Bilinear Quadrilateral Finite Element
3:15 - 3:35	Jonathan Beezley *CU - Denver*	Predictor-Corrector and Morphing Ensemble Filters
3:40 - 4:00	McKenna Roberts *CU - Denver*	Evaluation of Parameter Effects in Estimating Non-linear Uncertainty Propagation

Plenary Speaker (10:45 - 11:45)

THE NONUNIFORM FFT AND MAGNETIC RESONANCE IMAGE RECONSTRUCTION

Leslie Greengard, Courant Institute, New York University

The nonuniform FFT arises is a variety of applications, from medical imaging to radio astronomy to the numerical solution of partial differential equations. In a typical problem, one is given an irregular sampling of N data points in the frequency domain with the goal of reconstructing the corresponding function at N points in the physical domain. When the sampling is uniform, the fast Fourier transform (FFT) allows this calculation to be carried out in O(N log N) operations. Unfortunately, when the data is nonuniform, the FFT does not apply. In the last few years, a number of algorithms have been developed which overcome this limitation and are often referred to as nonuniform FFTs. In this talk, we describe the basic algorithm and its application to magnetic resonance imaging.

About the speaker: Dr. Leslie Greengard received his B.A. in mathematics from Wesleyan University in 1979, his Ph.D. in computer science from Yale University in 1987, and his M.D. from Yale University in 1987. During 1987 – 89 he was a NSF Postdoctoral Fellow at Yale University in the Department of Computer Science. He is presently professor of mathematics and the Director of the Courant Institute of New York University. Much of his work has been in the development of analysis-based fast algorithms such as the Fast Multipole Method for gravitation and electromagnetics and the Fast Gauss Transform for diffusion. Among the many awards he received for his work is the 2001 Steele Prize in Mathematics.

Morning Session I

ISING MODEL APPLIED TO CONGRESSIONAL REDISTRICTING - MCM 2007

Brandon Booth, Ben Safdi, and Kye Taylor
University of Colorado at Boulder
Room 1603, 9:00 - 9:20

We realize a novel method for determining fair, compact, and equal population Congressional districts by formulating an energy function whose ground state corresponds to the desired district plan. Our energy function is inspired by the Hamiltonian formulation for ferromagnets. We utilize statistical mechanics machinery and employ a Simulated Annealing algorithm to find near-ground states of our Hamiltonian operator. Two Hamiltonians are presented, which have a common population term and distinct compactness operators. To find a range of appropriate weights between compactness and population equality, we introduce an original search function capable of scanning through hundreds of Hamiltonians while continuously evolving the weight emphasis in real time. We apply our models to the state of New York and obtain exceptionally compact districts with populations varying less than a standard deviation away from the target mean. We propose a final district map for the state that we believe to be the most simple and fair as we define the terms.

THE KIDNEY EXCHANGE PROBLEM - MCM 2007

Sanghui Lee, John Villavert, and Nathan Schill
University of Colorado at Colorado Springs
Room 1603, 9:25 - 9:45

This year for the MCM/ICM competition, we chose problem C - the ICM problem. There are about 100,000 Americans today, that need an organ transplant, and over 70% of them are waiting on a kidney. For most of these patients, the only real solution is to register on the wait-list for a kidney transplant. However, there are many factors that dictate whether a patient can receive a certain kidney or not, such as compatibility, geography, and the present health condition of the patient. The tasks for this problem included modeling the present organ transplant system in place for two countries, and a way to "pair"patients with compatible donors. We tackled this problem by first making programs that "created" patients and donors, then matched them depending on the compatibility factors. Then we ran these programs to give us information on the estimated number of patients who might die before receiving a kidney. Our conclusions were not surprising, but reassuring.

AIRPLANE BOARDING MODEL - MCM 2007

Lee Rosenberg, Derlin Campbell, and Lydie Van Holland
University of Colorado at Denver
Room 1603, 9:50 - 10:10

Airlines are faced with a paradoxical situation with regard to the boarding of aircraft. On one margin, they are necessitated to form a more functional system of loading and unloading of the aircraft, yet on another, they risk the addition of change that could possibly mar the very operation of their trade. The problem given to us is that of modeling and optimizing this process in a quantitative manner. Developing a basic model of human behavior was the most excruciating portion of the endeavor. The basic model involved the use of random numbers and the concept of a queue to simulate human behavior. Having a basic model provided a means of comparison to existing research and a guide to perfecting the existing techniques. Perfecting the model was the absolute intent of the work. We introduced the idea of a scalar to represent the expedience of a given passenger, as well as a stepwise function for the stowage of carry-on baggage in the overhead bins. A discrete function involving distance, each passenger's boarding time, and other factors hindering the procession to a seat was utilized to simulate the abstract model of different airplanes. We devised a new method that airlines could implement that takes into account the many different variables that affect human behavior, such as age and air travel experience. This method uses the behavior scalar and sorts the passengers both by row number (rearmost to foremost row) and behavior scalar values. We devised a complex model for Southwest's random boarding, as well as the outside-in and rear to front methods. These simulations performed both to our specification and that of real world statistics. With respect to the new method that we formulated, we were able to take from 2.13 minutes up to 11.83 minutes off of the standard boarding times for a Boeing 737-300 aircraft.

STABILITY OF SOLUTIONS TO A REACTION DIFFUSION SYSTEM BASED UPON CHEMICAL REACTION KINETICS

Jisun Lim
University of Colorado at Boulder
Room 1603, 10:15 - 10:35

This talk explores stability to some mathematical models that describe chemical reaction kinetics. We analyze this property of a set of deterministic ordinary differential equations which is induced by one reversible chemical reaction mechanism containing three chemical species. We further discuss stability problem on a set of reaction diffusion equations based on the same chemical reaction. The results have shown that applying the the method of Liapunov, all solutions of the model are asymptotically stable. This allows us to find certain limits of concentration of each species as time proceeds.

Morning Session II

ELECTRICAL IMPEDANCE TOMOGRAPHY AND THE PIONEERING WORK OF A. CALDERON

Jutta Bikowski
Colorado State Universtiy
Room 1605, 9:00 - 9:20

The pioneering work on an inverse boundary value problem by A. Calderon has inspired a multitude of research, both theoretical and numerical, on the inverse conductivity problem. The problem has an important application in a medical imaging technique known as electrical impedance tomography (EIT) in which currents are applied on electrodes on the surface of a body, the resulting voltages are measured, and the inverse conductivity problem is solved to determine the conductivity distribution in the interior of the body, which is then displayed to form an image. This talk will give a short introduction to EIT and especially to Calderon and his ideas. This includes an overview of his article and we will show how the reconstruction method proposed by Calderon can be applied to simulated and experimental data, including perfusion data collected on a human chest. To our knowledge this is the first implementation of this method for experimental data and it yields promise as a fast and stable reconstruction method.

RECONSTRUCTIONS OF CONDUCTIVE AND INSULATING TARGETS USING THE D-BAR METHOD ON AN ELLIPTIC DOMAIN

Ethan Murphy
Colorado State University
Room 1605, 9:25 - 9:45

The D-bar algorithm based on A. Nachman's 2-D global uniqueness proof for the inverse conductivity problem (1996 Ann. Math. 143 7196) is implemented on an elliptic domain. The scattering transform is computed on an ellipse and the complete electrode model (CEM) for the forward problem is computed with the finite element method (FEM) in order to obtain static conductivity reconstructions of conductive and insulating targets in a saline-filled tank. It is demonstrated that the spatial artifacts in the image are significantly reduced when the domain is properly modeled in the reconstruction, as opposed to being modeled as a disk.

FLUID DYNAMICS IIN CYLINDRICAL GEOMETRIES: SIMPLIFYING NAVIER-STOKES FOR GEOPHYSICALLY RELEVANT PROBLEMS

Mike Watson
University of Colorado at Boulder
Room 1605, 9:50 - 10:10

Although the Navier-Stokes equations fully describe fluid motion, it is well known that both analytic and numeric solutions can be difficult to calculate even for relatively simply geometries and forcing. In geophysical fluid dynamics, we can simplify the governing equations by exploiting dominant forces and geometric constraints to introduce an asymptotic parameter. This asymptotic parameter then leads to a set of reduced equations, which capture the leading order dynamics of the fluid flow. In this talk, I will introduce a set a of reduced equations for radial heating in a tall aspect ratio annulus under rotation. These new equations can be used to analyze the baroclinic instability, which is one mechanism for large scale atmospheric events such as the jet stream and high/low pressure systems.

FINITE-TIME SINGULARITIES IN COUPLED NONLINEAR SCHRODINGER EQUATIONS WITH 4-WAVE MIXING

Benjamin Safdi
University of Colorado at Boulder
Room 1605, 10:15 - 10:35

New singular solutions are found in a set of coupled nonlinear Schrodinger (NLS) equations with 4-wave mixing. Unlike previous singularities in NLS systems, these do no imply spatial collapse but rather unbounded growth in wave amplitudes, with no spatial varia- tion. Moreover, a boundary is found in coefficient space such that singular solutions are guaranteed if the coefficients of the nonlinear terms satisfy a certain inequality. With no spatial variation, the coupled partial differential equations reduce to ordinary differential equations (ODEs). Then the condition on the coefficients allows an explicit general solution of the ODEs. The singularities are unavoidable and found to occur for all values of the Hamiltonian.

Afternoon Session I

ISOLATION AND IMPLEMENTATION OF THE DYNAMICAL CORE FROM THE GERMAN WEATHER SERVICE'S NUMERICAL WEATHER PREDICTION MODEL

Lauren Anderson
University of Colorado at Boulder
Room 1603, 1:00 - 1:20

Isolation of the dynamical core from the German Weather Service's numerical weather prediction model serves as an experimental framework for dynamical simulations up to 300 km in altitude. This requires an understanding of the primitive equations of meteorology, the numerical method for their solution and running the model on NCAR's parallel computers. From here, various physical parameterizations such as friction and radiation can be turned off, resulting in an isolated dynamical core that allows for extension of the upper boundary (currently near 30 km). Implementation of the Held-Suarez test serves as a test of the dynamical core by introducing an idealized forcing in place of the actual physics. This results in a mechanistic dynamical model that serves as a framework for testing the response of the atmosphere to various forcing, especially in the lower atmosphere. The model is also formulated for easy modification for multi-planet applications.

LOCAL ADAMPTIVE REFINEMENT (LAR)FOSLS

Josh Nolting
University of Colorado at Boulder
Room 1603, 1:25 - 1:45

Local refinement enables us to concentrate computational resources in areas that need special attention, for example, near steep gradients and singularities. In order to use local refinement efficiently, it is important to be able to quickly estimate local error. FOSLS is an ideal method to use for this because the FOSLS functional yields a sharp a posteriori error measure for each element. This talk will discuss a strategy for determining which elements to refine in order to optimize the accuracy per computational cost. Set in the context of a full multigrid algorithm, our strategy for the steep gradient case, leads to a refinement pattern with nearly equal error on each element. Further refinement is essentially uniform, which allows for an efficient parallel implementation. In the case where singularities exist, the refinement pattern is modified to account for sub-optimal reduction of the error in elements close to the singularity. Numerical experiments will be presented.

LOCAL GEOMETRIC PROJECTION FOR DENOISING M-VALUED DATA

Kye Taylor
University of Colorado at Boulder
Room 1603, 1:50 - 2:10

As high-dimensional data observed from physical systems is becoming more and more abundant, several nonlinear methods for "learning" the dataset's underlying topological structure have been proposed that rely on the spectral properties of a similarity matrix defined on a dataset, e.g. Laplacian Eigenmaps, Diffusions Maps, multi-dimensional scaling, and stochastic neighbor embedding just to name a few. When measured observations of an underlying signal are corrupted by noise and added dimensionality, performance of the algorithms may vary. Under the assumption that the data is derived from a dynamical system defined on a manifold embedded in a higher dimensional feature space (key to all techniques mentioned above), we first project data points onto an approximate tangent space, then denoise the data by reprojecting it back onto a physically realizable coordinate space, improving the data's SNR.

H-AVOIDING HAMILTONIAN CYCLES

Angela Harris
University of Colorado at Denver
Room 1605, 3:15 - 3:35

A spanning cycle in a graph G is called a hamiltonian cycle and if such a cycle exists, we say that G is hamiltonian. For a subgraph H of G we say that G is H-avoiding hamiltoninan if there is a hamiltonian cycle in G that contains no edges of H or, in other words, if G - E(H) is hamiltonian. In this talk we will discuss some new results on H-avoiding hamiltonian graphs. In particular we will consider the case where H is a hamiltonian cycle, which will lead to a discussion of extending families of edge-disjoint hamiltoinian cycles.

USING A GENERALIZED EIGENSOLVER BASED ON SMOOTHED AGGREGATION (GES-SA) TO INITIALIZE SMOOTHED AGGREGATION MULTIGRID

Geoffrey Sanders
University of Colorado at Boulder
Room 1603, 2:50 - 3:10

Consider the linear system Ax=b, where A is a large, sparse and symmetric positive definite matrix. Solving this system for unknown vector x using a smoothed aggregation multigrid (SA) algorithm requires a characterization of the algebraically smooth error that is poorly attenuated by the algorithm's relaxation process. For many relaxation processes of interest, algebraically smooth error corresponds to the near null-space of A. An eigenvector corresponding to the smallest eigenvalue of A, or minimal eigenvector, is an essential part of the near-nullspace of A. Therefore, having an approximate minimal eigenvector can be useful to characterize the algebraically smooth error when forming the SA method. This talk discusses the details of a generalized eigensolver based on smoothed aggregation (GES-SA) that is designed to produce an approximation to a minimal eigenvector of A. GES-SA might be very useful as a stand-alone eigensolver for applications that desire an approximate minimal eigenvector, but the intention here is to use GES-SA to produce an initial algebraically smooth eigenvector that can be used to initialize SA or an adaptive version of SA.

SOLUTION OF LARGE SPARSE UNDERDETERMINED LINEAR SYSTEMS WITH EMBEDDED NETWORK STRUCTURE FOR A NON-HOMOGENEOUS NETWORK FLOW PROGRAMMING PROBLEM

Eugene Vecharynski
University of Colorado at Denver
Room 1603, 3:15 - 3:35

The work on this paper was caused, mainly, by the analysis of non-homogeneous problems of network optimization on large data files. Our main goal was to develop an effective (direct) method for solving large sparse systems of linear equations with embedded network structure, which appear naturally, e.g. as systems of restrictions, in a broad class of nonhomogeneous network flow programming problems. The "network nature" of the regarded system allows keeping data in the matrix-free form in the computer memory. The formulae, derived within the paper, are written in the component (network) form to provide clear approaches towards developing computational algorithms using efficient data structures for graph representation.

AN INTRODUCTION TO SPECTRAL METHODS VIA THE NUMERICAL SOLUTION FO A FLUID TRANSPORT EQUATION

Keith Wojciechowski
University of Colorado at Denver
Room 1603, 3:40 - 4:00

This talk introduces spectral methods for numerically solving partial differential equations. In particular, it addresses how they can be applied to solving a partial-integral differential equation (PIDE) used to model moisture transport and viscoelastic stresses during sorption and drying of a porous medium. This PIDE takes the form of a diffusion equation plus a memory term which gets expressed as a Volterra-like integral operator acting on the temporal derivative of the independent variable. Numerically solving the PIDE poses problems because the memory term accumulates round-off error as the chosen numerical solver propagates in time. Thus, incorporating a spectral method makes a compelling choice because of its high order of accuracy. Emphasis will be placed on numerically solving the diffusion equation and constructing highly accurate differentiation matrices.

Afternoon Session II

THE IMPACT OF CONDITIONAL CASH TRANSFERS ON PRENATAL HEALTH: IS IT JUST A SUPPLY EFFECT?

Gisella Kagy
University of Colorado at Boulder
Room 1605, 1:00 - 1:20

This research provides a statistical analysis of the impact of Progresa, Mexico's principle anti-poverty program, on prenatal health. Progresa aims to end the intergenerational transmission of poverty by combining a traditional cash transfer program with requirements that participants invest in the human capital of their families. Our research utilizes the controlled random design of Progresa such that rigorous statistical analysis can be readily preformed. Specifically, we employ an ordinary least squares regression model to help determine which independent characteristics are statistically significant in regards to several dependent variables. Additionally, a multinomial logic model is used to model discrete dependent variables. From this investigation, we expect to find that Progresa has a significant positive impact upon prenatal healthcare in Mexico, regardless of healthcare supply changes throughout the program.

JUKES-CANTOR DNA PROBABILITY MODEL

Robert Engle
University of Colorado at Colorado Springs
Room 1605, 1:25 - 1:45

The observed fraction of substitutions q in an alignment between two sequences is a very important variable in any molecular evolution analysis. If an alignment suggests that relatively few substitutions have been done, then a simple count would suffice to determine the value of q. This means that if the strands are reasonably small, simply inspecting them visually for differences would work. However alignments with many differences could cause a significant underestimation of the number of substitutions. Even in small strands if there a significant number of substitutions then it can be unreasonable to count them visually or if the strands are very long then counting the substitutions would be near impossible. Jukes-Cantor derived a method of estimating the number of substitutions when simple inspection of two sequences was not possible. As a result of their derivation analysis of mitochondria DNA between humans and even species has become easier.

ILLUMINATION FACE SPACES ARE IDIOSYNCRATIC
Jen-Mei Chang
Colorado State University
Room 1605, 1:50 - 2:10

Illumination spaces capture how the appearances of human faces vary under changing illumination. This work models illumination spaces as points on a Grassmann manifold and uses distance measures on this manifold to show that every person in the CMU-PIE and Yale data sets has a unique and identifying illumination space. This suggests that variations under changes in illumination can be exploited for their discriminatory information. As an example, when face recognition is cast as matching sets of face images to sets of face images, subjects in the CMU-PIE and Yale databases can be recognized with 100% accuracy.

MATHEMATICAL AND NUMERICAL MODELING OF WILDFIRES
Minjeong Kim
University of Colorado at Denver
Room 1605, 2:25 - 2:45

The goal in wildland fire modeling is to predict the behavior of a complex system involving many processes and uncertain data by a physical model that reproduces important dynamic behaviors. A wildfire is modeled by a system of nonlinear reaction-convection-diffusion equations. Solution of such equations exhibit traveling waves, which consist of reaction zone and cool-down zone. The width of the reaction zone is less than the modeling scale and thus modeling in the reaction sheet limit is also considered. The model can be a convection dominated model or diffusion dominated due to various wind speeds in the wildland. It makes numerical difficulties and requires the efficient and accurate ways to solve under stability region. The level set method with adaptation can be applied to improve the numerical difficulties.

A NOTE ON THE PATCH TEST FOR THE BILINEAR QUADILATER FINITE ELEMENT
Bedrich Sousedik
University of Colorado at Denver
Room 1605, 2:50 - 3:10

The finite element method in 2D with bilinear shape functions and exact quadrature reproduces exactly a solution that has constant gradient. We show that this is the case also for the iso-parametric bilinear quadrilateral element on an arbitrary mesh and with 2 by 2 node Gaussian quadrature, when the stiffness matrix and the load vector are no longer exact.

PREDICTOR-CORRECTOR AND MORPHING ENSEMBLE FILTERS
Jonathan Beezley
University of Colorado at Denver
Room 1603, 2:25 - 2:45

New ensemble filter methods are proposed, which build on the concepts of traditional Ensemble Kalman Filters (EnKF) and particle filters. These new methods are designed to be more suitable in systems with a high degree of non-linearity and which tend to develop highly non-Gaussian forecast densities. The predictor-corrector filters in development first apply the EnKF to shift the ensemble near the location indicated by the data. The corrector phase then applies weights to each ensemble member to correct for any non-Gaussianity in the forecast distribution in a manner similar to a particle filter. These predictor-corrector filters are shown to combine the benefits of the EnKF and particle filters, while canceling out many of their problems. In the formulation, we have exploited the fact that model states tend to be solutions of differential equations and are elements of a space of smooth functions. This fact greatly reduces the effective dimensionality of the problem and allows the application of traditional particle filtering techniques. The morphing ensemble filters proposed attempt to remove the non-Gaussianity of the forecast distribution by transforming the ensemble state into something that is expected to be distributed more normally. For the purposes of transformation, each ensemble state variable is represented as a morphing function and a residual, which are computed using well known automated image registration techniques. The morphing function essentially describes the major features of the image in terms of their spatial location, while the residual describes any difference in the nature of the feature itself. Preliminary results for this method are given which show significant success in assimilating data with a poorly chosen initial ensemble, while eliminating filter divergence.

EVALUATION OF PARAMETER EFFECTS IN ESTIMATING NON-LINEAR UNCERTAINTY PROPAGATION

McKenna Roberts
University of Colorado at Colorado Springs
Room 1605, 3:40 - 4:00

The propagation of uncertainty in nonlinear cases can be handled accurately and easily with a piecewise linear approach to propagating the probability density function through the analysis equation. Previous work outlined this method but did not examine the effects of parameters on the accuracy of the results. Parameters to be chosen in this approach include the number of evaluation points and the distribution of those points over the range of interest. The effects of these parameters are explored for three elementary functions. It is found that for the functions examined, the piecewise linear approach always converged with increasing numbers of points. For the cases examined, 30-200 evaluation points were required for convergence. A uniform distribution of points was both the simplest to implement and converged the fastest.