# Program for the 2006 Front Range Applied Mathematics Student Conference

# Breakfast and Registration: 8:30 - 9:00

# Morning Session - Room 1521 9:00 - 10:30

9:00 - 9:20	Geoff Sanders University of Colorado, Boulder	Adaptive Non-Symmetric Smooth Aggregation Multigrid
9:25 - 9:45	Brad Klingenberg University of Colorado, Boulder	Exploiting the Geometry of Non-negative Matrix Factorization
9:50 - 10:05	McKenna Roberts University of Colorado, Colorado Springs	Modeling the External Counterpulsation Effects on the Human Arterial System
10:10 - 10:25	John Hyatt Colorado School of Mines	Domain Decomposition with Non-Matching Grids for Orthogonal Spline Collocation Problems

# MCM/ICM Session - Room 1525 9:00 - 10:20

9:00 - 9:15	Sanghui Lee (presenter), Joe Campillo, and Benjamin Hewett University of Colorado, Colorado Springs	Treating the Plague of the Twenty-First Century: A Projection of HIV Infection with Recommendations for Global Expenditures		
9:20 - 9:35	Caryn Knutsen, Kirsten Pyhtila, and Daniel Newman University of Colorado, Colorado Springs	Proposing Modern Efficiency for a Modern Airline		
9:40 - 9:55	Jeremy Noe, Michael Morrison, and Leah Grant University of Colorado, Denver	Analysis and Optimization of Hand-Move Sprinkler Systems		
10:00 - 10:15	Brian Camley, Pascal Getreuer, and Brad Klingenberg University of Colorado, Boulder	Sprinkle, Sprinkle Little Yard		
Break: 10:30 - 10:45				

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### Plenary Address, David Donoho: 10:45 - 11:45

Exotic New Data Need Exotic New Wavelets

## Lunch: 12:00 - 1:00

# Afternoon Session I - Room 1521 1:00 - 3:15

1:00 - 1:20	Darren Homrighausen University of Colorado, Denver	An Investigation into Statistical Tests for Stochastic Dominance with Applications to Economic Decision Theory
1:25 - 1:40	Michael Giannetto Colorado State University, Pueblo	Symplectic Integrators: We Know Good Approximation
1:45 - 2:05	Pascal Getreuer University of Colorado, Boulder	Nonlinear Multiresolution: New Discretizations for ENO Schemes
2:10 - 2:25	Brian Camley University of Colorado, Boulder	Modeling Polymerization In A Smectic Liquid Crystal Host With Random Trapping
2:30 - 2:50	Brendan Sheehan University of Colorado, Boulder	Spatial Multigrid for Transport
2:55 - 3:10	Jingling Guan University of Wyoming	Computational Simulation and Modification of Supersonic Delta Wings

# Afternoon Session II - Room 1525 1:00 - 3:15

1:00 - 1:20	Mikal Grant University of Wyoming	Numerical Simulations of Stochastic Differential Equations
1:25 - 1:40	Kristopher Tucker University of Colorado, Boulder	Improving How Search Engine Performance is Quantified
1:45 - 2:05	Rachel Danson University of Colorado, Boulder	A Proposed Method for Analyising EEG Data
2:10 - 2:25	Kye Taylor University of Colorado, Boulder	Automated Authorship Attribution Using Artificial Neural Networks
2:30 - 2:50	Brenda Christensen University of Wyoming	Identity Verification in Electronic Voting
2:55 - 3:10	Nathan Kurtz University of Colorado, Denver	Vertices of the Same Degree in Tournaments

# Plenary Speaker (10:45 - 11:45)

### EXOTIC NEW DATA NEED EXOTIC NEW WAVELETS David Donoho, Stanford University

Exotic new data types are constantly being developed and deployed in many fields of science and technology, and this causes exciting new challenges for statisticians and applied mathematicians. Data coming from navigation, motion capture, extragalactic astronomy, medical imagery, and other fields can be interpreted as manifold-valued data, i.e. as data where the individual data points, instead of being numbers on the real line, are points in a smooth manifold.

Examples include orientations (the manifold is a sphere) subspaces (the manifold is then the Grassmann manifold) and diffusion tensors (the manifold of positive definite matrices).

In the examples I will discuss, we have values in a manifold as a function of time and/or space. Thus for example, human motion is captured as a series of values in the manifold formed by the product of several Grassmann manifolds, one for each joint.

I will give numerous examples of manifold-valued data, stressing the intuitive nature of such data. I will describe some standard problems in signal processing, including compression, noise removal, enhancement, and explain why these are also important in the manifold-valued setting.

I will then discuss recent results providing multiscale 'wavelet' methods for dealing with such data. I will show how to use differential geometry to generalize existing notions of wavelets and will give novel examples of compressed, denoised, and contrast-enhanced human motion. The talk should be accessible to a broad audience with some mathematical background.

This is joint work with Inam Rahman and Iddo Drori of Stanford and Peter Schroeder (Caltech).

# MORNING SESSION

### ADAPTIVE NON-SYMMETRIC SMOOTH AGGREGATION MULTIGRID Geoff Sanders (Geoffrey.Sanders@colorado.edu) University of Colorado, Boulder Marian Brezina, Tom Manteufful (Advisor), Steve McCormick, and John Ruge

Algebraic Multigrid has been of interest for solving large linear systems that come from discretizing PDEs on unstructured meshes or complicated geometries. Adaptive algorithms have been devloped that use little or no a priori knowledge about the geometry of the problem. These methods tend to fail on systems that are very non-symmetric. This is a presentation of a developing algorithm that intends to expand the class of problems that Adaptive Smooth Aggregation Multigrid can handle by including many non-symmetric problems within it.

### EXPLOITING THE GEOMETRY OF NON-NEGATIVE MATRIX FACTORIZATION Brad Klingenberg (Bradley.Klingenberg@colorado.edu) University of Colorado, Boulder Advisors: James Curry and Anne Dougherty

With numerous applications to feature extraction, component analysis, and numerical linear algebra, non-negative matrix factorization (NMF) can provide insight into a variety of problems. Unfortunately, while several iterative algorithms for performing the factorization exist, convergence is slow. An investigation into the relevant geometry provides insights into the underlying problem. An algorithm which exploits the geometry of NMF is presented for a special case, and strategies for generalizing the method in a future work are discussed.

### MODELING THE EXTERNAL COUNTERPULSATION EFFECTS ON THE HUMAN ARTERIAL SYSTEM McKenna Roberts (mrobert2@uccs.edu) University of Colorado at Colorado Springs

Adequate modeling of the human arterial system is a central problem for physiology and medicine, both in normal and pathological conditions. We develop a compartmental model for the cardiovascular system in normal physiological conditions and also in presence of external factors, such as pressure cuffs designed to improve blood circulation. The code developed during this project simulates the effect of external counterpulsation (ECP) on the vascular system in its entirety, displaying the pressure and flow-rate at each compartment before, during and after the counterpulsation event. The workload of the heart (pressure-volume relationship) is measured during the experiment. This project was supported by a Colorado Institute of Technology grant.

### DOMAIN DECOMPOSITION WITH NON-MATCHING GRIDS FOR ORTHOGONAL SPLINE COLLOCATION PROBLEMS John Hyatt (jhyatt@mines.edu) *Colorado School of Mines* Advisor: Bernard Bialecki

We consider the development of methods for solving elliptic partial differential equations on a rectangle using domain decomposition with non-matching grids for orthogonal spline collocation(OSC). The domain is divided into nonoverlapping subdomains and OSC is used to obtain numerical solutions on each subdomain. In general, one subdomain solution cannot be obtained independently of the other subdomain solutions. Hence, the principal difficulty in applying domain decomposition methods is in the application of interface constraints. We consider two different sets of interface constraints, patching and mortar.

We first develop methods for solving Poissons equation on a rectangle that is decomposed into two nonoverlapping subdomains with non-matching grids. The grids under consideration are uniform on each subdomain, and the grid size of one subdomain is an integer multiple of the grid size of the other subdomain. The patching method requires continuity of the subdomain solutions at the interface collocation points from one subdomain, and continuity of the normal derivatives of the subdomain solutions at the interface collocation points of the other subdomain. We reduce the patching OSC problem to a problem on the interface. We solve this interface problem, and then solve OSC problems independently on each subdomain. Numerical results indicate that the error convergence is suboptimal. We reformulate the problem using mortar conditions on the interface. Mortar conditions are generally used in conjunction with finite element methods. These conditions consist of integral constraints on the subdomain solutions. Our approach is a discrete mortar method in the sense that we use a discrete inner product instead of integrals. Additionally, we include constraints on the normal derivatives of the subdomain solutions. We reduce the discrete mortar OSC problem to an interface problem. We solve this interface problem, and then solve OSC problems independently on each subdomain. Numerical results indicate that this method gives optimal error convergence.

We proceed to develop an efficient method for solving the discrete mortar OSC problem. This efficient method consists of three steps. The first and the third steps require solving OSC problems independently on each subdomain. Efficient methods for these steps appear in the OSC literature and the cost is  $O(N^2 \log N)$ , where N is the number of subintervals in the coarse grid. The second step consists of solving the interface problem. In the reduction to the interface problem, we introduce a Steklov-Poincare operator. This Steklov-Poincare operator provides a natural choice for a preconditioner which we use in conjunction with the generalized minimal residual(GMRES) iterative procedure. The cost of the second step is  $O(mN \log N)$ , where m is the number of GMRES iterations. In our numerical examples m is proportional to  $\log N$  and the total cost of solving the problem is  $O(N^2 \log N)$ . We briefly discuss an extension to problems with differential operators that have varying coefficients. We outline an efficient method for solving the problem and present numerical results.

# MCM/ICM SESSION

### TREATING THE PLAGUE OF THE TWENTY-FIRST CENTURY: A PROJECTION OF HIV INFECTION WITH RECOMMENDATIONS FOR GLOBAL EXPENDITURES Sanghui Lee (presenter), Joe Campillo, Benjamin Hewett University of Colorado, Colorado Springs

In twenty-five years, AIDS has taken twenty-two million lives, created more than thirteen million orphans, and is currently infecting sixteen thousand new people daily (Sax, 3). To assist in fighting this plague, our team modeled the spread of HIV/AIDS infections for six nations, one on each populated continent, through 2050. This report will use the six countries estimates of HIV spread as controls, comparing them with adjusted models that incorporate the effect of a hypothetical AIDS vaccine and antiretroviral therapy (ARV), used singly or in combination. We will also model the effect of ARV resistant strains appearing. With this data, the report will illustrate the effectiveness of one or both options and, show the areas in which the worlds money would best be put to use. Our model estimates the number of people who will be infected each year from 2006 to 2050 by employing an algorithm that estimates the number of babies being born with HIV, the number of people actively spreading HIV and their success rate, and accounting for a reduction in the HIV population due to deaths. A high GDP is used as a reduction factor in the HIV birth estimates. ARV treatments and vaccines are used in modeling the adult spread and ARV treatments reduce the death rate. The main strength of our model is that we take many variables into account, including each the social practices within the HIV population in each country that may increase or decrease the chances of HIV transmission. A weakness of the model is that we take into account GDP growth, but not a cost inflation rate. We also assigned values to parameters that cannot be calculated or measured, in order to approximate the effectiveness of treatments.

#### PROPOSING MODERN EFFICIENCY FOR A MODERN AIRLINE

#### Caryn Knutsen, Kirsten Pyhtila, and Daniel Newman University of Colorado, Colorado Springs

Acknowledging that Epsilon Airlines complies with all Federal Aviation Administration regulated rules concerning disability services; our team proposes an algorithm that optimizes the efficiency of airline escorts and wheelchairs, which in-turn minimizes the monetary loses of the airline. We have found that based on our algorithm total costs would range from 342,960to571,580 for the present year, and that the Airline will have a specific number of escorts needed based on high and low traffic times for each of their gates in small, medium, and large concourses. Furthermore, using this model to observe probable upcoming events, our model proves to be a resource for not only the present, but also for the future. Our model showed that average yearly costs due to wages and wheel chair maintenance will range from 752, 160to1,629,680 in the year 2016. Also, the number of escorts needed is accurate as we recalculated the most important perceived variables such as population growth. The unlimited number of outside variables and needed statistics proved to be a difficult task to create a strict model. As a result, a fair amount of logical assumptions were required. Nonetheless, the end result does not impinge on the usefulness of the algorithm, how the algorithm is programmed, or how the algorithm works. Moreover, based on our positive results our model should be considered for acceptance to Epsilon Airlines and should be exploited without delay to begin maximizing profit and time.

### ANALYSIS AND OPTIMIZATION OF HAND-MOVE SPRINKLER SYSTEM Jeremy Noe, Michael Morrison, and Leah Grant University of Colorado, Denver

To address the issue of irrigating a 30 by 80 meter field, our team of analysts meticulously considered the utility of a 20-meter hand-move pipe irrigation system with respect to three distinctive sprinkler head arrangements. We explored realistic expectations for water velocity, wetting area, and evaporation, and these formed several parameters for possible configurations. We propose our models accordingly, with the additional objectives of maximizing efficiency and minimizing cost.

At the outset, we offered our individual interpretations of the problem and collaborated to arrive at a simplistic model. We considered developing this basic workable representation of the problem to be our main focus. Once established, we evaluated the technical aspects to use for improvement of future layout scenarios.

Having accomplished this, we focused our subsequent efforts on achieving complete field coverage. This was the primary consideration of our second attempt. After coming up with a new layout, we analyzed the technical aspects set forth in our first model and determined that this was not our best option.

Using what we learned from constructing the first two models, we vastly improved upon several aspects of efficiency in our final sprinkler layout. Incorporating the assumption of a non-uniform water distribution, we were able to most accurately analyze the coverage and determine watering duration. This allowed us to recommend a very practical scheduling algorithm.

Our optimal solution proposes that the field be divided into four sections of equal area, each 20 by 30 meters in dimension. The irrigation system is oriented across each quadrant, perpendicular to the 30-meter edges. We suggest that the system remains stationary during the course of a fifteen-hour watering period; this requires the least amount of time and energy expenditure since the pipe need only be moved once on a daily basis.

### SPRINKLE, SPRINKLE LITTLE YARD Brian Camley, Pascal Getreuer, and Brad Klingenberg University of Colorado, Boulder

We determine an optimal algorithm for irrigating an 80 m x 30 m field using a hand-move 20 m pipe set with a combination of analytical arguments and simulated annealing. We minimize the number of times the pipe is moved and maximize the Christiansen uniformity coefficient of the watering.

We describe water flow from a sprinkler with the physical model of fluid flow from a pipe combined with projectile motion with air resistance; this predicts a range and distribution consistent with data from the literature. This profile allows us to develop a mathematical model of hand-move pipe irrigation. We analytically determine the position of sprinkler heads on a pipe to optimize uniformity of watering, and show that our analytic results are consistent with predictions by both simulated annealing and Nelder-Mead optimization.

Using an averaging technique inspired by radial basis functions, we prove that periodic spacing of pipes maximizes uniformity. Numerical simulation supports this result; we construct a sequence of irrigation steps and show that both the uniformity and number of steps required are locally optimal. We show that in order to prevent overwatering, we cannot leave the pipe in a single location until the minimum watering requirement for that region is met. In order to water sufficiently we must water in several passes, noting that the number of passes is minimized as uniformity is maximized. We propose watering the field with four repetitions of five steps, each step lasting roughly thirty minutes. We place only two sprinkler heads on the pipe, one at each end. The five steps are uniformly spaced from along the long direction of the field with the first step at the field boundary. The pipe locations are centered in the short direction. This strategy requires only twenty steps and has a Christiansen uniformity coefficient of 94.4, well above the commercial irrigation minimum of 80. Running a simulated annealing experiment re-creates our solution from a random initialization when we maximize uniformity of watering.

The consistency between the solutions generated by numerical optimization and analytical techniques suggests that our result is accurate and at least a local optimum. Moreover, the solution remains optimal upon varying the sprinkler profile, indicating the results are not overly sensitive to our initial assumptions. The robustness and consistency of the solution serves as a compelling validation of our models and general approach.

# AFTERNOON SESSION I

### AN INVESTIGATION INTO STATISTICAL TESTS FOR STOCHASTIC DOMINANCE WITH APPLICATIONS TO ECONOMIC DECISION THEORY Darren Homrighausen (dwhomrig@ouray.cudenver.edu) University of Colorado, Denver David Kahle (University of Richmond) and Dr. Javier Rojo (Rice University)

Stochastic Dominance (SD) is an ordering relation on the space of random variables that allows for a probabilistic ranking to be established. In areas of application such as income distributions or portfolio theory, this corresponds to orderings in terms of poverty and welfare or investment returns, respectively. Most tests for stochastic dominance either approximate the ecdfs with known distributions or make comparisons at only arbitrary fixed points. These are not desireable features as they add considerably to test inconsistency. Using an estimator developed by Barrett and Donald that is based on the Kolmogorov-Smirnov test, it is possible to check for  $j^{th}$  order stochastic dominance by comparing distributions at every point of their support. In the case of first order SD, the test statistic converges to a Brownian Bridge Process, thus allowing for asymptotic critical values can be derived. However, it is unclear how well these critical values perform for smaller sample sizes. Additionally, it is not possible to find a closed form expression for higher order SD.

Through the use of various Monte Carlo methods, we simulate critical values of the test statistic for first, second, and third order SD. These critical values allow for an alternative method of testing for SD. Also, having critical values permits us to check the power and probability of type one error for the test using various distributions and sample sizes.

SYMPLECTIC INTEGRATORS: WE KNOW GOOD APPROXIMATION Michael P. Giannetto (giannetto69@hotmail.com) Colorado State University-Pueblo Advisor: Dr. Bruce Lundberg A Symplectic integrator is a numerical method that approximates the solution of a Hamiltonian system of differential equations. Symplectic integrators better approximate solutions of orbital systems over the long term. Good approximation depends upon choosing an efficient step sizea trade off between speed and accuracy. Symplectic integrators are designed to preserve certain differential invariants like phase space volume. Since angular momentum and energy are not preserved, it is the goal of our research to compare symplectic integrators with general purpose integrators using these physical properties. These provide a measure of the physical accuracy of computed solutions.

### NONLINEAR MULTIRESOLUTION: NEW DISCRETIZATIONS FOR ENO SCHEMES Pascal Getreuer (getreuer@colorado.edu) University of Colorado, Boulder

Essentially Non-Oscillatory (ENO) schemes are nonlinear multiresolution decompositions designed to efficiently represent piecewise smooth data. ENO schemes are usually applied with either point-value or cell-average discretization. This presentation will show how to construct ENO and Harten schemes consistent with Harten's framework for a variety of discretizations. The construction here begins with the discrete operators and deduces the corresponding continuous operators, reversing the order of the usual approach. As a special case of this construction, ENO for any order of spline discretization is developed. This approach also has the flexibility to define schemes with non-spline discretizations, allowing for an extension to a variety of ENO schemes.

### MODELING POLYMERIZATION IN A SMECTIC LIQUID CRYSTAL HOST WITH RANDOM TRAPPING Brian Camley (camley@colorado.edu) DEPARTMENT OF PHYSICS University of Colorado, Boulder Advisor: Leo Radzihovsky

Hybrids between ferroelectric liquid crystals and polymers are exciting materials that combine interesting fundamental physics with a high potential for a variety of applications, such as flexible, ultra-fast liquid crystal displays and artificial muscles. We use a combination of numerical and analytical tools to explore the dynamics of a simplified model (proposed by Radzihovsky and Clark) that we expect captures the essential physics of the polymerization in the smectic host. We describe the rate of polymerization by treating diffusing monomers as point particles in the presence of traps, which represent polymerization locations. This idealization has applications to a large variety of physical problems, such as electron motion in the presence of impurities and particle localization in a random potential. as well as being of pure mathematical interest. We use generating functions, statistics of random walks, and simple scaling arguments to describe both the short-time monomer concentration and the variance in concentration. We also treat the probability of trapping when the motion of monomers is impeded in one direction (e.g., by the smectic layers). Results are compared with Monte Carlo simulations and experimental data, showing good agreement.

#### SPATIAL MULTIGRID FOR TRANSPORT Brendan Sheehan (Brendan.Sheehan@colorado.edu) University of Colorado, Boulder Advisor: Tom Manteuffel

A spatial Multigrid algorithm for isotropic neutron transport is presented in x-y geometry. The problem is discretized with discrete ordinates in angle and corner balance finite differencing in space. Spatial smoothing is accomplished by a four color block jacobi relaxation, where the diagonal blocks correspond to four cell blocks on the spatial grid. A bi-linear interpolation operator and its transpose are used for the grid transfer operators. Encouraging preliminary results are presented for homogeneous domains. Heterogeneous domains are also discussed, especially the case of a vacuum region surrounded by a diffusive region.

### COMPUTATIONAL SIMULATION AND MODIFICATION OF SUPERSONIC DELTA WINGS Jingling Guan (guanjingling@hotmail.com) University of Wyoming

This research focuses on computational simulation of airflows around delta wings at supersonic speed. The research consists of a simulation stage and a modification stage. In the simulation stage, grids are generated and transformed, and then a numerical method is set up to solve the governing PDE. The solutions of the PDE shows how exactly the airflow behaves when an aircraft is traveling supersonic at different angles of attack and also with different shapes of delta-wings, ranging from conical to elliptic.

In the modification stage, by changing the gas law inside the sonic bubble found in the simulation stage, smoother sonic bubbles can be found. Then by setting up a new coordinate system inside the sonic bubble, solving a Cauchy problem, and tracing the boundary, the modified wing shape, which will produce the smoother sonic bubble, can be found. Thus, this computational methodology can be used to find the optimal shape of delta wings that minimizes shockwaves.

# AFTERNOON SESSION II

### NUMERICAL SIMULATIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS Mikal K. Grant (GODFATHR@uwyo.edu) University of Wyoming

This talk includes an overview of stochastic differential equations, basic methods used in creating models/simulations of these equations using computers, and applications of these models in various ways ranging from economics to combustion.

#### IMPROVING HOW SEARCH ENGINE PERFORMANCE IS QUANTIFIED Kris Tucker (Kristopher.Tucker@colorado.edu) University of Colorado, Boulder

Information retrieval (IR) is currently a topic of great interest in both the computer science and applied mathematics communities. As such, there is a great demand for a means of quantifying search engine performance that allows for quick and convenient evaluation of IR algorithms. The currently accepted convention of determining search engine performance through a comparison of recall and precision can be drastically improved upon by utilizing statistical measures of agreement. The measure of agreement known as Cohens kappa is especially well suited to applications in IR. First, it offers a single scalar quantity to describe performance, thus eliminating the ambiguity that may arise as a result of using two values like recall and precision while allowing for the use of general optimization routines. Its most striking feature, however, is its ability to more accurately reflect the performance of IR algorithms by neglecting to take into account features of the data that arise due to chance alone.

### A PROPOSED METHOD FOR ANALYZING EEG DATA Rachel Danson (Rachel.Danson@colorado.edu) University of Colorado, Boulder

The electroencephalograph (EEG) is widely used by cognitive psychologists to explore which frequencies of brain waves are associated with certain tasks. Typically the subject is exposed to a stimulus such as a picture, word, or sound. The EEG records the brain waves before and after the stimulus. In any one EEG recording from a subject there is noise, masking the underlying brain activity associated with the stimulus. To reduce this noise multiple trials are conducted by displaying the same stimulus to the subject. The raw data from the trials associated with the stimulus of interest are then averaged together creating an Event Related Potential (ERP).

The EEG data before the stimulus is assumed to be uncorrelated with the stimulus and therefore merely ongoing brain activity. When the EEG trials are averaged, these data are assumed to cancel each other out. The post-stimulus ERP usually displays a waveform that is associated with the stimulus. Though ERPs are often used in current psychological research, it is unknown whether the resulting waveform after the stimulus is due to an increase in amplitude of ongoing brain activity at particular frequencies, a form of phase reset inside the brain, or a combination of both.

To identify if the ERP waveform is produced by phase resetting a new method is being developed. First a Morlet wavelet is applied to the raw EEG data for each of the trials. This shows the frequency bands that are primary contributers to the signal at a given time. For each trial, frequency, and time step the phase is then computed. Using directional statistics, the variance of the phases is computed. According to the literature the phases before the stimulus are expected to be uncorrelated. In this case being uncorrelated translates to the prestimulus data having a higher variance than the post variance data. Using the pre-stimulus variances as a control group, it can determined if the post stimulus phase variances are significantly different from the pre-stimulus variances. This talk will present the results of research work done to date and will discuss future work in this area.

### AUTOMATED AUTHORSHIP ATTRIBUTION USING ARTIFICIAL NEURAL NETWORKS Kye Taylor (Kye.Taylor@colorado.edu) University of Colorado, Boulder Advisor: M.O. Tearle

Neuro-computing provides a novel opportunity to investigate the disputed or unknown origins of literary works that until recently could receive only subjective review in deciding authorship. Results are obtained using a "Committee of Machines" where each committee member is a multi-layer perceptron (MLP) trained to differentiate between works of different authors based on stylometric parameters that are assumed to be indicative of an author's style. Different variables that quantitatively measure word collocations, proportionate pairs, words per sentence, letter spacing, *etc.*, are used as inputs to each network. The networks are then trained on known works of contempories such as William Shakespeare and Thomas Marlowe before making a decision on an unknown or disputed work by one of these authors.

In designing each network in the committee, m different discriminators on the literature were measured. Assuming all m are helpful in classifying any given sample, each neural network would consist of m input neurons, a hidden layer of n neurons and finally an output layer of a single neuron corresponding to either a match or not between the work being investigated and a given author. The neurons in each layer are connected by different weights which are determined via successful training of the network. Random initial weights can lead to any number of different trained networks in the committe capable of correctly identifying the known test works, analogous to different viewpoints of human committee members.

A robust network relies on the ability to generalize from the limited training data. It is a necessary axiomatic claim that there exists a non-linear function to determine authorship of a given sample; it is, however, a further claim that the m discriminators chosen as network inputs are in fact the independent variables of that function. Nonetheless, the network performs well in most cases. However, in problematic samples corresponding either to an incorrect classification or to no consistent classification from the committee, questions surrounding the ability of the networks to generalize to new data, due to the high dimensionality and limited size of the training data, become paramount. Methods in exploratory statistical analysis and data mining are used to optimize the variables and network configuration used in construction of the MLP, given that linear correlation may likely not be observable in the traditional sense when considering a nonlinear, multi-dimensional, binary image function from limited data.

### IDENTITY VERIFICATION IN ELECTRONIC VOTING Brenda Christensen (BRENDA23@uwyo.edu) University of Wyoming Advisor: Bryan Shader

Electronic voting is right on the horizon, but many complications stand in the way. One of the major obstacles faced by electronic voting is identity verification it is critical that both parties involved are in fact who they should be. As the voter, you want to make sure that you are in fact logged onto the correct sight and giving your information to a secure site. From the perspective of the program, you want to know that the person trying to submit the vote is only the individual entitled to that vote. This speech focuses on how the identities of the parties involved can be verified through secure means.

#### VERTICES OF THE SAME DEGREE IN TOURNAMENTS Nathan Kurtz (nkurtz@math.cudenver.edu) University of Colorado, Denver Mike Ferrara, Mike Jacobson

Let T be a tournament of order n. It is easy to see that every two vertices of the same degree in T lie on a directed cycle of length 3. In this talk we look at the problem of deciding whether two vertices of the same degree lie on a  $C_l$  given that T contains a  $C_l$ . We show if T is strong then every set of k vertices of the same degree lie on a  $C_k$  and/or a  $C_{k+1}$  as well as a  $C_{k+2}, \dots, C_n$ . We also consider which cycles contain the arc xy, where x and y have the same degree in T.