

Write your name below. This exam has 5 problems and is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use one page of notes. You cannot collaborate on the exam or seek outside help, nor can you use a calculator or any computational software.

Name: _____

Do not begin until instructed.

1. (15 points) Consider the three vectors:

$$\mathbf{v}_1 = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

$$\mathbf{v}_2 = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$

$$\mathbf{v}_3 = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

- (a) (5 points) What is the volume of the parallelepiped formed by these three vectors?
- (b) (10 points) Calculate the following. If an expression does not exist, indicate this by writing "DNE".

i. $(\mathbf{v}_1 - \mathbf{v}_2) \times \mathbf{v}_3$

ii. $(\mathbf{v}_1 \cdot \mathbf{v}_2) \times \mathbf{v}_3$

iii. $(\mathbf{v}_1 \cdot \mathbf{v}_2)\mathbf{v}_3$

iv. $\mathbf{v}_2 \times (\mathbf{v}_3 \times \mathbf{v}_1) + \mathbf{v}_2 \times (\mathbf{v}_1 \times \mathbf{v}_3)$

2. (20 points) A line is given by the symmetric equations

$$\left(\frac{x-3}{2}\right) = \left(\frac{y-2}{3}\right) = (z-1)$$

- (a) (5 points) What is the vector equation for this line?
- (b) (5 points) How far is the point $P(1, 2, 3)$ from the line?
- (c) (10 points) Find the point where the line above intersects the line given by

$$\mathbf{r}(t) = \langle 1, 2, 3 \rangle + \langle 1, 2, 3 \rangle t$$

If the lines do not intersect, indicate whether they are parallel or skew.

3. (25 points) Consider the plane given by

$$3x - 2y + z = 2$$

and the line given by the symmetric equations

$$\frac{x+1}{2} = y = 2-z.$$

- (a) (5 points) Find the point where the line intersects the plane.
- (b) (5 points) Find the angle between the line and plane's normal vector.
- (c) (5 points) Show that the point $A(3, 2, 0)$ is on the line.
- (d) (10 points) Determine how far the point A is from the plane.

4. (25 points) A quadric surface is defined by the equation

$$2x^2 - 4x + y^2 + 4y - z^2 = 0$$

- (a) (7 points) Write the equation of the surface in standard form.
 - (b) (4 points) Classify the surface and state its orientation. What is the center of the surface?
 - (c) (7 points) Sketch the trace for $x = 1$. Label all y and z intercepts.
 - (d) (7 points) Sketch the trace for $z = 6$. Label all x and y intercepts.
5. (15 points) An ion is moving in a magnetic field along the space curve given by

$$\mathbf{r}(t) = \frac{1}{\sqrt{2}} \langle \cos(t), \sqrt{2} \sin(t), \cos(t) \rangle$$

- (a) (10 points) Find the velocity vector, $\mathbf{r}'(t)$, and acceleration vector, $\mathbf{r}''(t)$ of the ion. What is their dot product?
- (b) (5 points) When $t = \frac{\pi}{4}$, the magnetic field is switched off, causing the ion to move in a straight line. Find an equation for this line.