

1. (12 pts) Consider the rational function

$$r(x) = \frac{3x^2 + 7}{(x-2)(x^2-4)(x^2+x+9)^2}.$$

Write out the form of the partial fraction decomposition, but DO NOT solve for the coefficients.

2. (36 pts) Evaluate the following integrals:

(a) $\int (2 \sin \theta + \cot \theta)^2 d\theta$

(b) $\int x \ln(x+1) dx$

(c) $\int \frac{\sqrt{x^2+4}}{x^4} dx$ *Hint: $\tan \theta$ and $\sec \theta$ can be expressed in terms of $\sin \theta$ and $\cos \theta$.*

3. (20 pts) Determine whether the following integrals are convergent or divergent. If the integral converges, find its value; if you use the Comparison Test, state this and fully justify it.

(a) $\int_6^\infty \frac{(1 + \frac{1}{x})}{\sqrt{x^2 - x - 3}} dx$

(b) $\int_0^2 \frac{1-x}{\sqrt{x}} dx$

4. (16 pts) Consider the table below, which provides values of $f(x)$ and some of its derivatives. You may also assume that $f''(x)$ is decreasing for all $x \geq 0$.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$f(x)$	3	$\frac{1}{2}$	$-\frac{1}{9}$	$-\frac{3}{7}$	$-\frac{2}{3}$	$-\frac{3}{4}$	$-\frac{4}{5}$
$f''(x)$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$-\frac{1}{5}$	$-\frac{12}{5}$	-3	$-\frac{7}{2}$

- (a) Write out an approximation for $\int_1^3 f(x) dx$ using the Trapezoidal Rule with $n = 4$. *You DO NOT need to simplify your answer, but it should be an expression which is easy to enter into a calculator.*
- (b) Give a range of n -values where the trapezoidal rule would give an approximation of $\int_1^3 f(x) dx$ with an error less than or equal to 10^{-3} . *Again, you do not need to simplify your answer, but it should be an expression which is easy to enter into a calculator.*
5. (16 pts) Determine if the following statements are **True** or **False**, and give a short justification for your answer. If you believe (a) or (b) are false, give a specific example which shows that the statement is false.

(a) If $\int_{-\infty}^{\infty} f(x) dx$ converges, then both $\int_{-\infty}^1 f(x) dx$ and $\int_1^{\infty} f(x) dx$ must converge as well.

(b) Suppose $a(x)$ and $b(x)$ are positive functions with $a(x) \leq b(x)$ on $[1, \infty)$. If $\int_1^{\infty} a(x) dx$ converges, then $\int_1^{\infty} b(x) dx$ does as well.

(c) The function $f(x) = \frac{2x^2 + \sqrt{x} - 1}{(x+2)(x-1)(x^2+4)}$ has a decomposition given by

$$\frac{A}{x+2} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}.$$

(d) Suppose $x = 3 \sec \theta$. Then $\sin \theta \tan \theta = \frac{x^2 - 9}{3x}$.