21st Annual SIAM Front Range Applied Mathematics Student Conference

March 8th, 2025

Check-in, Welcome and Breakfast: 8:30 - 9:00

Student Commons Building, Room 4128

Morning Session I: 9:00 - 11:00

Student Commons Building, Room 4125

9:00 - 9:20	Devlin Costello *University of Colorado, Boulder*	Properties and Limitations of Source Localization via Laplace Transforms
9:25 - 9:45	Gunnar Enserro, Austin Long, and Phoenix Williams *University of Colorado, Denver*	Exploring Critical Behavior in Neurodynamics of Motion through the Ising Model
9:50 - 10:10	Kristina Moen *Colorado State University*	Texture Analysis of Satellite Imagery for Weather Forecasting
10:15 - 10:35	Chris Rocheleau *Colorado State University*	Anatomically-Informed EIT Reconstruction for Pulmonary Function in Preterm Infants
10:40 - 11:00	Lillian Makhoul *University of Colorado, Denver*	Finishing the chapter: computing the convex hull of the graph of a trilinear monomial over a general box domain

Morning Session II: 9:00 - 11:00

Student Commons Building, Room 4017

9:00 - 9:20	Jacob Dunham and Andrew Kitterman *University of Colorado, Denver*	A Discussion on Strategies for Symmetric Group Solitaire Games
9:25 - 9:45	Jerry Wang, Matthew Struckenbruck, and Tony Samour *University of Colorado, Boulder*	A "Step-By-Step" Walkthrough on the Wearing of Stairs (MCM)
9:50 - 10:10	Andrew Kitterman *University of Colorado, Denver*	Improving the Accuracy of POD-Based Surrogates
10:15 - 10:35	Jerry Wang *University of Colorado, Boulder*	Modulation Evolution Along Multi Soliton Solutions of the KP II Equation
10:40 - 11:00	Sean Svihla *University of Colorado, Boulder*	Sparsification of Phylogenetic Covariance Matrices of Critical Beta-splitting Random Trees

University of Colorado Denver
CU IN THE CITY

We gratefully acknowledge funding support from CU Denver Dept. of Mathematical and Statistical Sciences and the J.R. Woodhull/Logicon Teaching Professor of Applied Mathematics at CU Boulder.

Coffee Break: 11:05 - 11:30

Industry Panel: 11:30 - 12:30
North Classroom, Room 1130

Panelists:

- Tonya Allen, **Principal Staff Program Manager, SAFe/Agile Coach** (*ServiceNow*)
- Richie Clancy, **Senior Principal Mathematician** (*Northrop Grumman*)
- Paulo Burke, **Senior Bioinformatics Software Engineer** (*Children's Hospital Colorado*)
- Michael Schmidt, **Principal Machine Learning Engineer** (*Redpoll*)

A group picture will be taken immediately following the industry panel.

Lunch Break: 12:30 - 1:15
North Classroom, Room 1130

Plenary Address: 1:30 - 2:30
North Classroom, Room 1130

"Imaging lung function in infants using electrical impedance tomography"

Dr. Jennifer Mueller. *Professor, Department of Mathematics, Colorado State University*

Afternoon Session I: 2:45 - 3:55

Student Commons Building, Room 4125

2:45 - 3:05	Abigail Nix *University of Colorado, Denver*	The Diameter of Partition Polytopes
3:10 - 3:30	John Small *University of Wyoming*	Numerical Analysis of a (Class of) Problem(s) of Ordinary Differential Equations on a Manifold
3:35 - 3:55	Krithikesh Ravishankar *University of Colorado, Boulder*	Reservoir computing based control of chaotic systems

Afternoon Session II: 2:45 - 3:55

Student Commons Building, Room 4017

2:45 - 3:05	Michael Schmidt *University of Colorado, Denver*	Hierarchical Dirichlet Processes for Global Sensitivity Analysis
3:10 - 3:30	Nic Rummel *University of Colorado, Boulder*	WENDy in Julia

Afternoon Session III: 2:45 - 3:55

Student Commons Building, Room 4113

2:45 - 3:05	Kyler Howard *Colorado State University*	3D EIT Reconstructions of Ventilation and Perfusion from Dual-Patch Electrode Array Data: Preliminary Results
3:10 - 3:30	Trevor Overton *Colorado State University*	Implementing a Schur Complement based post processing technique for Electrical Impedance Tomography using data collected on infants
3:35 - 3:55	Zachary Sorenson *University of Colorado, Denver*	A Cycle-Canceling Heuristic for the Directed Traveling Salesman Problem

Industry Panel

11:30 - 12:30

Tonya Allen

Principal Staff Program Manager, SAFe/Agile Coach
ServiceNow

Tonya Allen is a Principal Staff Program Manager and the sole Agile Coach at ServiceNow, where she empowers leaders, teams, and stakeholders to enhance their effectiveness, culture, and value delivery through Lean-Agile principles. With over 20 years of experience in IT and software engineering, Tonya has led large-scale, multi-million-dollar software development programs at ServiceNow, TiVo, and Level3 Communications. She is a graduate of the Colorado School of Mines with a degree in Mathematics and Computer Science. Beyond her professional work, Tonya is deeply committed to family, community, and advocacy for equity and justice. A Colorado native and mother of two teenagers, she enjoys traveling, hiking, camping, road biking, snowboarding, scuba diving, yoga, and volunteering.

Richie Clancy

Senior Principal Mathematician
Northrop Grumman

As a mathematician at Northrop Grumman, I focus on algorithmic development for target tracking and sensor fusion applications. I received my PhD in Applied Mathematics from CU-Boulder in 2022 with an emphasis on optimization and inverse problems. Prior to graduate school, I worked as a quantitative analyst and natural gas options trader.

Paulo Burke

Senior Bioinformatics Software Engineer
Children's Hospital Colorado

I currently develop data pipelines for genomic data at the Precision Diagnostics Lab at Children's Hospital Colorado. I've completed my master's in computer science and my PhD in bioinformatics at University of Sao Paulo, in Brazil, studying complex systems with a focus on biochemical and other biological networks. That background empowered me to advance data processing at scale in a clinical genomics environment, impacting lives of thousands of patients.

Michael Schmidt

Principal Machine Learning Engineer
Redpoll

Michael Schmidt is a Principal ML Engineer at Redpoll and a doctoral candidate at UC Denver. He specializes in Bayesian non-parametric methods, sensitivity analysis and change point detection. He has worked on several DARPA projects concerning agential learning and novelty detection. His current work focuses on convergence diagnostics and distributed learning. In his spare time, he likes to climb and ski.

Plenary Speaker

1:30 - 2:30

Jennifer Mueller

Professor, Department of Mathematics
Colorado State University

IMAGING LUNG FUNCTION IN INFANTS USING ELECTRICAL IMPEDANCE TOMOGRAPHY

Electrical impedance tomography (EIT) is a relatively new medical imaging technique in which electric fields are used to form real-time images of organ function and structure. It is particularly suitable for imaging infants since it does not impart ionizing radiation. To form these images, it is necessary to solve a severely ill-posed inverse problem with computational efficiency. This talk will provide an introduction to EIT and the state-of-the art for pulmonary imaging. Direct (non-iterative) and iterative reconstruction methods, including the D-bar method, a direct reconstruction algorithm based on techniques of inverse scattering, will be presented. Results from data collected on infants at Children's Hospital Colorado will demonstrate its potential for monitoring infants in the NICU.

About the Speaker

Dr. Jennifer Mueller is the Albert C. Yates Chair Professor of Mathematics and a Professor of Biomedical Engineering at Colorado State University (CSU). She received her Ph.D. from the University of Nebraska-Lincoln in 1997, and was an NSF Postdoctoral Fellow at Rensselaer Polytechnic Institute before joining the faculty at CSU in 2000. She held the 3-year position of Professor Laureate in the College of Natural Sciences at CSU from 2020 through 2022. Her research encompasses development of new hardware, reconstruction algorithms, and clinical applications for electrical impedance tomography (EIT) and ultrasound computed tomography (USCT). Supported by the NIH, her work involves close collaborations with mathematicians, engineers, and physicians in the U.S. and abroad. Dr. Mueller serves as an Associate Editor for IEEE Transactions on Medical Imaging and is the co-founding editor of the journal Applied Mathematics for Modern Challenges. She is co-author with Samuli Siltanen of the book "Linear and Nonlinear Inverse Problems with Practical Applications", SIAM, 2012, and they are working on a new book: "Introduction to Inverse Problems: A Data Science Perspective".

MORNING SESSION I

PROPERTIES AND LIMITATIONS OF SOURCE LOCALIZATION VIA LAPLACE TRANSFORMS

Devlin Costello
University of Colorado, Boulder

Infection spreading is a classical topic in mathematical biology and network theory. However, very little work has been done on the inverse problem: where did the infection originate? In this talk, we discuss how to localize the source of an infection spreading over a tree according to an SI model, i.e., nodes infect their susceptible neighbors after random delays. The challenge is to estimate the source only using the infection times of a proper subset of nodes called "observers." Using the theoretical and sample Laplace Transform of these infection times under each possible source, we develop a method with an implicit data augmentation to estimate the true source accurately. We discuss empirical results about simulated infections and theoretical guarantees surrounding how the method should behave.

EXPLORING CRITICAL BEHAVIOR IN NEURODYNAMICS OF MOTION THROUGH THE ISING MODEL

Gunnar Enserro, Austin Long, and Phoenix Williams
University of Colorado, Denver

The Ising Model or pairwise maximum entropy model has long been established as a valid quantitative simplification for capturing the collective behavior of networks of neural activity (Hopfield 1982 & 1984, Amit 1989, Hertz et al 1991). More recent work has shifted the interest in modeling neural activity towards verifying the ability for biological systems to be ”tuned” or ”self-tuned” towards criticality (Friedman et al 2012, Fontenele et al 2019) and proposed that the ”thermodynamical” behavior of the Ising model can capture critical behavior equally well to dynamical analysis of neural avalanches without having to define the parameters of avalanche events (Bialek and Meshulam 2024). However, interpreting the significance of relationships between thermodynamical quantities at critical points and neurodynamics has not been explored yet. In this presentation, we present findings on how the critical energy and critical temperature relate to anatomical data observations for neural activity networks of various cell groups within mice.

TEXTURE ANALYSIS OF SATELLITE IMAGERY FOR WEATHER FORECASTING

Kristina Moen
Colorado State University

Weather satellites above the Earth's surface give an ever-changing view of the atmospheric events that trigger severe weather conditions like storms, flash floods, tornadoes, and hurricanes. Due to the large amount of data available today, forecasters use automated tools to correctly identify events visible in a satellite images. Such tools are often created using neural networks, which require large labeled data sets that are simply not available for many weather events. On the other hand, forecasters can reliably identify events by recognizing changes in texture. We present several texture-based methods that distinguish between convective and non-convective clouds in a set of Geostationary Operational Environmental Satellite (GOES) images. We discuss how texture-based methods can be used in tandem with machine learning methods that can then be trained on a small number of labeled samples but still identify complex weather events.

ANATOMICALLY-INFORMED EIT RECONSTRUCTION FOR PULMONARY FUNCTION IN PRETERM INFANTS

Chris Rocheleau
Colorado State University

Electrical impedance tomography (EIT) is a non-ionizing bedside medical imaging technique for computing real-time images of organ function of a patient based on voltage data arising from current applied on electrodes attached to the body. Due to the severe ill-posedness of the inverse problem, good spatial resolution poses a challenge in EIT. However, the temporal resolution is high, facilitating dynamic bedside imaging. In this talk, we propose the use of an anatomical atlas constructed from a database of CT scans of 89 infants to provide prior spatial information to the reconstruction algorithm. The MEan Atlas NOSER-based (MEAN) algorithm modifies the NOSER algorithm to utilize a non-constant initial estimate of an anatomically relevant distribution of conductivity and susceptivity. The updated initial estimate is constructed from an averaging of an anatomical atlas of EIT images and corresponding voltage estimates. We apply the method to data collected on preterm infants and healthy control infants to reconstruct conductivity and susceptivity images of both ventilation and perfusion in real time using the ACT 5 EIT system.

FINISHING THE CHAPTER: COMPUTING THE CONVEX HULL OF THE GRAPH OF A TRILINEAR MONOMIAL OVER A GENERAL BOX DOMAIN

Lillian Makhoul
University of Colorado, Denver

Algorithms for solving complex mixed-integer nonlinear optimization problems rely on obtaining good convex approximations of the non-convex sets that arise from substructures in the problem formulation. A common example is the graph of a trilinear monomial, $y = x_1x_2x_3$, defined over the box $[a_1, b_1] \times [a_2, b_2] \times [a_3, b_3] \subseteq \mathbb{R}^3$. By computing the volume of the convex hull, we obtain a measure that can be used to evaluate other convex outer approximations in comparison.

In previous work, Speakman and Averkov (2019) used a mixed-volume method to obtain a formula for the volume of the convex hull of the graph of a trilinear monomial in terms of the box parameters, however, the result is only valid when each of the bounds are nonnegative. Here, we perform a complete case analysis for general variable bounds and extend the mixed-volume methods to obtain volume formulae for each possible instance. We demonstrate that by relaxing the requirement that our variable bounds are nonnegative, the structure of the resulting convex hull polytope, and therefore its volume formula, is altered.

MORNING SESSION II

A DISCUSSION ON STRATEGIES FOR SYMMETRIC GROUP SOLITAIRE GAMES

Jacob Dunham and Andrew Kitterman
University of Colorado, Denver

The symmetric group S_n has an interesting solitaire game which expands on the notion of the Klein four-group to larger n. We present an approach to playing the game on S_n where $n \equiv 0 \bmod 8$, and ideas on how graphs can be used for other symmetric group solitaire games.

A "STEP-BY-STEP" WALKTHROUGH ON THE WEARING OF STAIRS (MCM)

Jerry Wang, Matthew Struckenbruck, and Tony Samour
University of Colorado, Boulder

For the 2025 COMAP mathematical modeling competition, we chose to analyze the pattern of wear on stone steps in archaeological remains with the aim of determining information about past usage such as frequency, direction and footstep location. To determine users' patterns of activity, we first used a continuous form of Archard's Law to derive the 3D mesh of the change in height on the step based on a convolution of the distribution of where footsteps would be located with the amount of pressure at each point a single footstep provides. Then, using an algorithm for inverting this convolution under some reasonable assumptions, the original distribution of footsteps could be derived from the 3D mesh, revealing information about the direction of travel and where and how the stairs were climbed by the inhabitants.

Furthermore, we developed two approaches for determining an approximation for the usage of the staircase, as quantified by the number of steps. The first of which utilized Paris' Law, an ODE for crack length growth with respect to load cycles. The other approach applied dimensional analysis in tandem with Archard's Law to find the lost staircase volume caused from sliding wear. Lastly, we conducted a brief analysis to further characterize the staircases, determining a rough age for the steps and accounting for the possibility that some steps had been repaired, or even replaced.

IMPROVING THE ACCURACY OF POD-BASED SURROGATES

Andrew Kitterman
University of Colorado, Denver

Gappy Proper Orthogonal Decomposition (POD) is a widely used surrogate modeling technique for reconstructing high-resolution data from incomplete observations. In a multifidelity setting, data is available at multiple levels of accuracy and computational cost, and leveraging this information effectively can improve surrogate model performance. We present two methods to enhance Gappy POD in this context: one that incorporates multifidelity data to improve prediction accuracy and another that provides useful estimates of modeling error. These approaches offer valuable insights into the reliability and effectiveness of reduced-order models in multifidelity scenarios.

SPARSIFICATION OF PHYLOGENETIC COVARIANCE MATRICES OF CRITICAL BETA-SPLITTING RANDOM TREES

Sean Svihla
University of Colorado, Boulder

Recent research has shown that, with high probability, a discrete analog of the Haar wavelet can nearly fully sparsify phylogenetic covariance matrices of large, uniformly random binary trees. However, while sparsification offers computational benefits and scientific insights, this model does not accurately capture notable features of real-world phylogenetic trees. This talk explores a more realistic model—the Critical Beta-Splitting Random Tree—introduced by D. Aldous in 1996 for this purpose. Through a detailed probabilistic analysis of the external path length in large trees from this ensemble, we demonstrate that the Haar-like basis almost fully sparsifies the phylogenetic covariance matrix for most large trees in this framework, highlighting our results' relevance and broader impact.

MODULATION EVOLUTION ALONG MULTI SOLITON SOLUTIONS OF THE KP II EQUATION

Jerry Wang
University of Colorado, Boulder

The Kadomtsev-Petviashvili II (KP II) equation is a partial differential equation that describes the evolution of weakly two-dimensional shallow water waves. The KP II Equation admits a variety of soliton solutions that represent localized nonlinear waves. In addition to line soliton solutions, KP II also admits more complex multi-"legged" soliton structures, such as Y- and X-shaped patterns. Using a previously developed modulation theory, the evolution of perturbations to Y-solitons are investigated. Mathematically, this involves posing and solving an initial-boundary value problem for the modulation of soliton parameters of each leg. The boundary conditions relate the parameters of each leg to the other legs at the intersection or vertex of the legs. Of particular interest is the propagation of disturbances through the soliton vertex. Analytic solutions are obtained for the linearized equations and some special nonlinear cases.

AFTERNOON SESSION I

THE DIAMETER OF PARTITION POLYTOPES

Abigail Nix
University of Colorado, Denver

In this talk, we discuss the combinatorial diameter of partition polytopes, polytopes associated with partitions of n items into k clusters, each of prescribed size. We define clustering difference graphs (CDGs) and use them to improve a known upper bound on the diameter of partition polytopes. Our new bounds are found using a graph theoretical construction, which will be demonstrated during the talk.

NUMERICAL ANALYSIS OF A (CLASS OF) PROBLEM(S) OF ORDINARY DIFFERENTIAL EQUATIONS ON A MANIFOLD

John Small
University of Wyoming

In this presentation, we will describe numerical approximations of a class of first-order initial value systems on a manifold in $\mathbb{R}^n$, characterized by a certain constraint. The main challenge in the typical numerical schemes is their inability to maintain the constraint. For a problem with a constant constraint, we found that a midpoint rule scheme is able to preserve the constraint irrespective of the time step, while Backward Euler and trapezoidal rule schemes only seem to obey the constraint asymptotically with respect to time step.

RESERVOIR COMPUTING BASED CONTROL OF CHAOTIC SYSTEMS

Krithikesh Ravishankar
University of Colorado, Boulder

This project investigates the control of an unknown chaotic dynamical system using reservoir computing, aiming to stabilize the system by mitigating parameter-driven disturbances. The model-free approach to build the controller only makes use of previous observations of the system, ensuring a robust methodology with wide ranging applications to real-world chaotic systems. The system's chaotic nature is influenced by disturbances to the system parameters themselves, effectively modifying the underlying model, causing it to dip into periodic or chaotic regimes. To address this, we apply reservoir computing alongside two control schemes: a delayed control scheme and a feedback control scheme and evaluate the conditions under which each scheme suppresses chaotic disturbances and maintains a periodic orbit. We found that the feedback control scheme is efficient and robust to large disturbances, and the delayed control scheme determines the disturbances exactly, thereby identifying the cause of the disturbances. This work offers insights into real-time control and chaos suppression, demonstrating the potential of reservoir computing and hybrid control frameworks to manage nonlinear dynamical systems in chaotic environments.

AFTERNOON SESSION II

HIERARCHICAL DIRICHLET PROCESSES FOR GLOBAL SENSITIVITY ANALYSIS

Michael Schmidt
University of Colorado, Denver

Understanding the connection between the uncertainty in a model's inputs and outputs is essential in model design. Not only is it necessary to understand the model's correctness, but it is also useful as an instrument to improve its design; if we know what drives uncertainty, we can aim to reduce it. Practitioners can understand input uncertainty's impact on output uncertainty using Global, momentum-independent sensitivity analysis (GMISA). In GMISA, the input and output variables are conceptualized as random variables X_i ($1 \leq i \leq n$ variables) and Y, respectively. The analysis's product is a sensitivity index representing the input distribution's expected impact on the output distribution, i.e., $p(Y|X_i)$ vs $p(Y)$. Such an analysis requires a robust density estimation of the joint distribution $p(X_i, Y)$ from a sample (x_i, y).

Conventionally, practitioners use distributional fitting or kernel density estimates (KDE). In this work, we investigated the robustness of KDE methods and proposed an alternative method to determine the GMISA sensitivity indices with density estimation via Hierarchical Dirichlet Processes (HDP). A HDP allows us to create a sample of distributions likely to have generated our sample (x_i, y). We find HDP more faithfully reproduces multimodal distributions and, as an added benefit, naturally leads to a distributional sample of each sensitivity index. We show that the HDP method reproduces the KDE-based method's performance and exceeds it in the unimodal case while more robustly representing the sensitivity indices in the multimodal case. In addition, we show that the uncertainty of sensitivity indices can share insights for model design.

WENDY IN JULIA

Nic Rummel
University of Colorado, Boulder

The Weak-form Estimation of Non-linear Dynamics (WENDy) algorithm is extended to accommodate systems of ordinary differential equations that are nonlinear-in-parameters. The extension rests on derived analytic expressions for a likelihood function, its gradient and its Hessian matrix. WENDy makes use of these to approximate a maximum likelihood estimator based on optimization routines suited for non-convex optimization problems. The resulting parameter estimation algorithm has better accuracy, a substantially larger domain of convergence, and is often orders of magnitude faster than the conventional output error least squares method (based on forward solvers).

The algorithm is efficiently implemented in Julia. We demonstrate the algorithm's ability to accommodate the weak form optimization for both additive normal and multiplicative log-normal noise, and present results on a suite of benchmark systems of ordinary differential equations. In order to demonstrate the practical benefits of our approach, we present extensive comparisons between our method and output error methods in terms of accuracy, precision, bias, and coverage.

AFTERNOON SESSION III

3D EIT RECONSTRUCTIONS OF VENTILATION AND PERFUSION FROM DUAL-PATCH ELECTRODE ARRAY DATA: PRELIMINARY RESULTS

Kyler Howard
Colorado State University

Electrical Impedance Tomography (EIT) is a medical imaging technique in which low-amplitude, low-frequency current is applied to electrodes placed around the body. Conventional 3D EIT images are collected with two electrode belts placed around the body. To detect breast cancers, dual-patch electrode arrays were introduced as an analogy to mammography. Due to their ease of application, the use of dual-patch arrays is of interest for pulmonary data collection in the ICU. We aim to investigate its suitability through data measured on a human torso and FEM simulations of human torso data. We demonstrate that FEM simulations can produce voltages in good agreement with the measured data and comparable differences in voltages between full inspiration and full expiration with the human data. Reconstructions of the simulated and measured data show clear visual changes between inhalation, exhalation, systole, and diastole, and for simulated ventilation of the left and right lung. These preliminary results suggest that the dual-patch electrode geometry is suitable for detecting single-lung ventilation and changes due to pulsatile perfusion. As shown in the single-lung ventilation cases, the dual-patch array may also detect various pathologies in a subject.

IMPLEMENTING A SCHUR COMPLEMENT BASED POST PROCESSING TECHNIQUE FOR ELECTRICAL IMPEDANCE TOMOGRAPHY USING DATA COLLECTED ON INFANTS

Trevor Overton
Colorado State University

Electrical Impedance Tomography (EIT) is a nonionizing, noninvasive pulmonary imaging technique for bedside monitoring. Functional images of conductivity are formed by measuring voltages that arise on electrodes from applied current patterns. Due to the severe ill posedness of the inverse problem, spatial resolution is one of the primary challenges in the field. One approach to improve resolution is to apply a statistical post processing method that uses the Schur complement to include information from an anatomical atlas of simulated EIT data. In this work, an anatomical atlas was constructed from CT scan data of infants. A computationally efficient Schur Complement method was implemented and applied to a verification data set to measure accuracy. Results showed significant improvement in resolution in various evaluation metrics.

A CYCLE-CANCELING HEURISTIC FOR THE DIRECTED TRAVELING SALESMAN PROBLEM

Zachary Sorenson
University of Colorado, Denver

The Traveling Salesman Problem (TSP) is one of the most studied problems in optimization. Since it is an NP-Hard problem, there are several algorithms in the literature that find locally optimal solutions, or tours. The purpose of this paper is to present a new heuristic that can be used in conjunction with these algorithms to improve locally optimal solutions.

Since some of the steps of this heuristic are similar to the cycle cancelling algorithm and the patching algorithm, we refer to it as the cycle canceling and patching heuristic (Cycap). The Cycap initially forms a set of graphs which contain cycles and circulations that are guaranteed to form either a tour or a set of subtours. As verified by several computational experiments, this heuristic is frequently able to improve tours. The average tour improvement made, when possibly significant, was significant, and the running times of the heuristic were fast enough to be viable in practice.