- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/112024 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The vector field $\mathbf{V}(x, y, z) = (z - x)\,\mathbf{i} + (z - y)\,\mathbf{j} + 2z\,\mathbf{k}$ is both incompressible and irrotational.

   (b) If $x = \dfrac{u}{v}, y = \dfrac{v}{w}, z = \dfrac{w}{u}$, then $\dfrac{\partial(x, y, z)}{\partial(u, v, w)} = 0$.

   (c) $\displaystyle\int_{-1}^{0}\int_{0}^{1+y} e^{x+y}\,\mathrm{d}x\,\mathrm{d}y + \int_{0}^{1}\int_{0}^{1-y} e^{x+y}\,\mathrm{d}x\,\mathrm{d}y = \int_{0}^{1}\int_{x-1}^{1-x} e^{x+y}\,\mathrm{d}y\,\mathrm{d}x$

   (d) If $\mathcal{C}$ is the curve $y = x^2, 0 \le x \le 1$, then the area under the graph of $f(x, y) = x + 2y$ above $\mathcal{C}$ is $\displaystyle\iint_{\mathcal{C}} (x + 2y)\,\mathrm{d}S$.

   (e) For any sufficiently differentiable vector field $\mathbf{F}, \nabla \times (\nabla \cdot \mathbf{F})$ is a meaningful expression.

2. [2350/112024 (20 pts)] On a certain day the density of lightning strikes in the Colorado Rocky Mountains was found to be well-approximated by the function

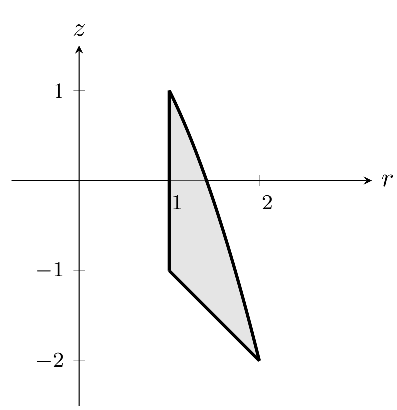
$$\delta(x, y) = \left(x^2 + y^2\right)^{3/2}\left[1 + 12\tan^{-1}(y/x)\right]$$

strikes per square mile. Some colleagues of yours need to calculate the total number of lightning strikes, N, in a certain area that day to help determine the risk of new forest fires. They have decided that this can be accomplished by computing the value of

$$N = \int_{\sqrt{3}/2}^{1}\int_{\sqrt{1-x^2}}^{x/\sqrt{3}} \delta(x, y)\,\mathrm{d}y\,\mathrm{d}x + \int_{1}^{\sqrt{3}}\int_{0}^{x/\sqrt{3}} \delta(x, y)\,\mathrm{d}y\,\mathrm{d}x + \int_{\sqrt{3}}^{2}\int_{0}^{\sqrt{4-x^2}} \delta(x, y)\,\mathrm{d}y\,\mathrm{d}x$$

However, since they took Calculus 3 many moons ago, they cannot seem to recall how to do this calculation. Come to their assistance by determining the total number lightning strikes in their area of interest, simplifying your final answer. Hint: Draw the region of integration and consider using another coordinate system.

3. [2350/112024 (20 pts)] The portion of a solid metal machine part is shown in a constant θ-plane in the figure below. The curved portion (on the right) is given by $x^2 + y^2 + z = 2$. The other boundaries are straight lines. If the weight density of the metal is $\delta(x, y, z) = \dfrac{6y}{x^2 + y^2}$ pounds per cubic inch, find the weight of that portion of the metal part lying between the planes $y = x$ and $y = -x$ with $y \ge 0$.



MORE PROBLEMS BELOW/ON REVERSE

4. [2350/112024 (8 pts)] Convert $\displaystyle\int_1^2 \int_0^{\pi} \int_0^{z/\sqrt{3}} r^3 \,\mathrm{d}r \,\mathrm{d}\theta \,\mathrm{d}z$ to spherical coordinates using the order $\mathrm{d}\theta \,\mathrm{d}\rho \,\mathrm{d}\phi$. **Do not** evaluate either integral.

5. [2350/112024 (22 pts)] In the written homework, you were asked to find the volume of the solid region bounded by $f(x, y) = 4 - x^2 - 2y^2$ and the xy-plane. This resulted in a fairly lengthy calculation. We will do this problem here in a much easier way.

   (a) [6 pts] Find $\mathcal{D}$, the region in the xy-plane over which you will integrate to compute the volume.

   (b) [10 pts] Use the change of variables $u = x, v = \sqrt{2}y$ to write a new integral that will give the volume, but **do not evaluate** it.

   (c) [6 pts] Use another (familiar) change of variables to find an integral that computes the volume and **evaluate** it.

6. [2350/112024 (20 pts)] The metal roof of a storage shed is described by the function $3z = 1 - x^2,\ z \geq 0,\ 0 \leq y \leq 2$. A wire runs along the edge of the roof where $y = 2$. In both of the following parts, you will set up an integral that computes the given quantity. Your final answer will be an integral simplified completely so that the only remaining step is to actually perform the integration, which you will **not** do.

   (a) [10 pts] The total charge, Q_r, on the roof if the charge density is $q(x, y, z) = xyz$ coulombs per square meter.

   (b) [10 pts] The total mass, m_w, of the wire if the mass density is $\rho(x, y, z) = 3xyz$ kilograms per meter.