Write your name below. This exam is worth 100 points. You must show all your work to receive credit on each problem and must fully simplify your answers unless otherwise instructed. You are allowed to use one page of notes. You are not allowed to use a calculator or any computational software.

Name:

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1. (26 points) Let $f(x, y) = \ln(9 - x^2 - 4y^2)$

   (a) (5 points) Sketch the domain of f, labeling all relevant points.

   (b) (6 points) Find a nonzero vector that is orthogonal to the level curve of f passing through the point $(2, 1)$.

   (c) (3 points) Find a vector pointing along the level curve at the same point.

   (d) (6 points) What is the rate of change of $f(x, y)$ at the point $(1, 1)$ in the direction of the vector $4\mathbf{i} + 5\mathbf{j}$?

   (e) (6 points) Let $x(s, t) = 3s\cos(t)$ and $y(s, t) = \frac{3}{2}s\sin(t)$. Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

2. (22 points) Let $f(x, y) = 3xy - x^2y - xy^2$.

   (a) (8 points) Find all the critical points of $f(x, y)$.

   (b) (8 points) Classify the critical points as local maxima, local minima, or saddle points using the second derivative test.

   (c) (6 points) Let T be the triangular region bounded by

$$\begin{aligned} x &= 0 \\ y &= 0 \\ y &= 2 - x \end{aligned}$$

   Find the extreme values of $f(x, y)$ on T.

3. (12 points) Find the following limits or show that they do not exist.

   (a) (6 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2(y)}{x^4 + 2y^4}$

   (b) (6 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^5 + y^3}$

4. (16 points) A long straight railroad tunnel has a cross section in the shape of a gaussian: $y = e^{-x^2}$. Use the method of Lagrange Multipliers to find the width and height of the largest railroad cars that can fit through the tunnel.

5. (24 points) Let $f(x, y) = e^{xy}$

   (a) (6 points) Find the tangent plane to the function near the point $(1, 2)$.

   (b) (8 points) Find the degree 2 Taylor approximation of $f(x, y)$ at the same point. You may leave your answer in factored form.

   (c) (10 points) What is the maximum error of the degree 1 Taylor approximation when $|x - 1| < 0.1$ and $|y - 2| < 0.1$?