

1. (28 pts) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Be sure to fully justify your answers.

(a)  $\sum_{n=0}^{\infty} \frac{(n+1)!}{(n!)^2}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{3^{2n}}{(n+1)^n}$

(c)  $\sum_{n=2}^{\infty} \frac{\ln n}{n^{5/2}}$

2. (28 pts) Let  $f(x) = \frac{5}{1 - \frac{x}{2}}$ .

- (a) Find a power series representation for  $f(x)$ . Simplify your answer.  
 (b) What is the radius of convergence of the power series?  
 (c) Does the power series converge at the endpoints of the interval of convergence? Justify your answer by writing out the endpoint series and determining whether they are absolutely convergent, conditionally convergent, or divergent.  
 (d) Find a power series representation for  $x^3 f'(x)$ . Simplify your answer.  
 (e) What is the sum of the series found in part (d)? Simplify your answer.

3. (20 pts) The function  $g(x)$  has the Maclaurin series representation  $\sum_{n=0}^{\infty} \binom{-1/2}{n} \frac{2}{3^n} x^{n+2}$ .

(For this problem, your answers should fully simplify all binomial coefficients.)

- (a) Approximate the value of  $g(1)$  using  $T_3(x)$ , the 3rd order Taylor polynomial for  $g(x)$ . Simplify your answer.  
 (b) Use the Alternating Series Estimation Theorem to find an error bound for the approximation found in part (a). You may assume that the conditions of the theorem are satisfied.  
 (c) Find a closed form (non-series) expression for  $g(x)$ .

4. (12 pts) The following two problems are not related.

- (a) Does the sum of the series exist? If so, find the sum. If not, explain why not.

$$\frac{10^1}{0!} - \frac{10^3}{2!} + \frac{10^5}{4!} - \frac{10^7}{6!} + \cdots$$

- (b) The function  $h(x)$  has the Taylor series representation  $\sum_{n=1}^{\infty} \frac{n^3(x-3)^{n-1}}{2^{n+1}}$ . Find the value of the

9th derivative of  $h(x)$  evaluated at  $x = 3$ . Leave your answer unsimplified.

5. (12 pts) Match each pair of parametric equations to one of the graphs shown below. No explanation is required.

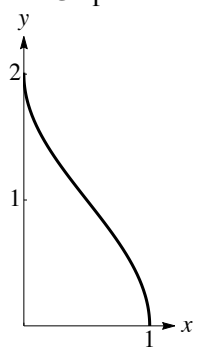
(a)  $x = 1 - \ln t$        $y = 2(\ln t)^2$      $1 \leq t \leq e$       Graph \_\_\_\_

(b)  $x = \sin t$        $y = 2 \cos t$      $0 \leq t \leq \frac{\pi}{2}$       Graph \_\_\_\_

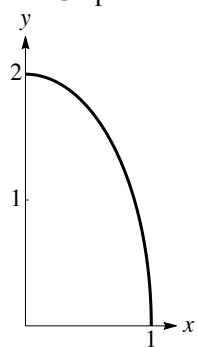
(c)  $x = \cos^2 t$        $y = 2 \sin^2 t$      $0 \leq t \leq \frac{\pi}{2}$       Graph \_\_\_\_

(d)  $x = -1 + \sec^2 t$      $y = 2 \tan^2 t$      $0 \leq t \leq \frac{\pi}{4}$       Graph \_\_\_\_

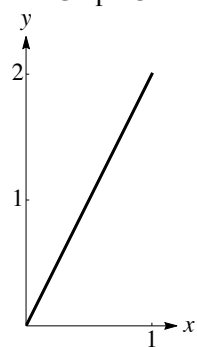
Graph A



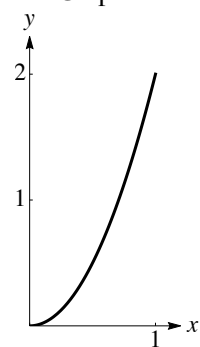
Graph B



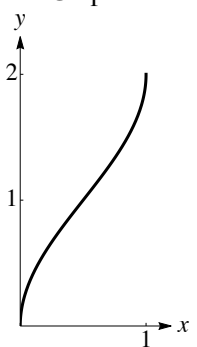
Graph C



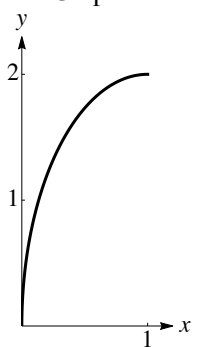
Graph D



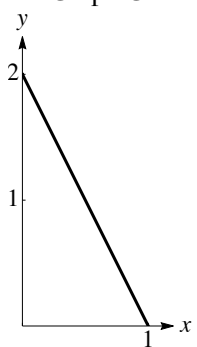
Graph E



Graph F



Graph G



Graph H

