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1. (30 points) The following problems are unrelated.

   (a) Evalaute $\displaystyle\int \left[\sqrt{x^5} + \frac{4x^8}{3ax^{2/3}}\right] dx$ where a is a constant.

   (b) Evaluate $\displaystyle\int_0^{\pi/4} \frac{\sec^2 x}{(5-2\tan x)^3}\, dx$.

   (c) Find the derivative of $y = 4x^3 + \displaystyle\int_{\sin x}^{x^2} \frac{5t+7}{\sqrt{8-3t}}\, dt$. (Please do not attempt to simplify your final answer.)

2. (22 points) Consider the following limit of a Riemann Sum:

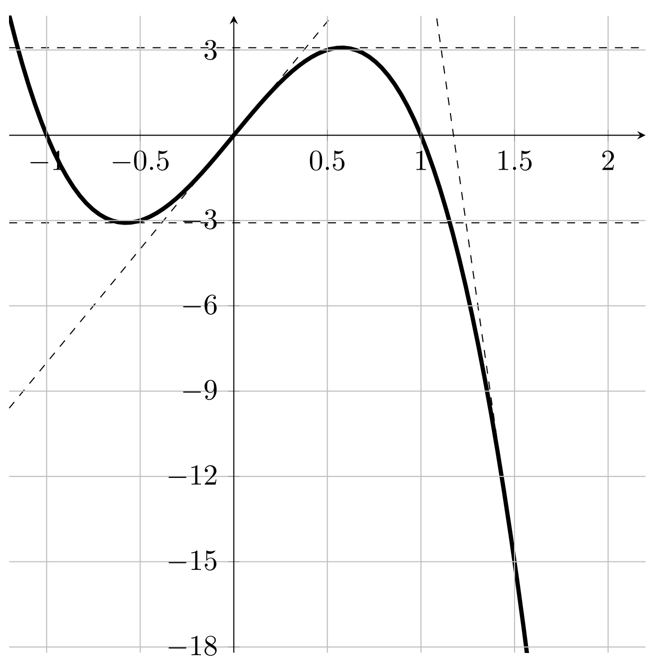
$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^2 + 1\right] \frac{2}{n}.$$

   (a) Evaluate the limit by first using summation properties and formulas to simplify the sum and then using the methods of chapter 1 to evaluate the limit.

   (b) Determine a definite integral, $\displaystyle\int_a^b f(x)\, dx$, that corresponds to the limit of the provided Riemann sum.

   (c) Find the average value of $f(x)$ over the interval $[a, b]$, where $f(x)$, a, and b, refer to the function and limits of integration you found in (b). (Note that you may use your answer from (a) to help find the solution to this problem.)

3. (20 points) Consider the graph of $y = g(x)$ provided below. The dashed lines represent the tangent lines of $y = g(x)$ at $x = \pm\frac{1}{\sqrt{3}}, 0$, and $\frac{3}{2}$. Use this graph to answer the questions that follow.



(a) Approximate $\int_{-1}^{3/2} g(x)\,dx$ using the right endpoint rule with five rectangles of equal width.

(b) Is the function $h(x) = \int_0^x g(t)\,dt$ concave up or concave down on $\left(0, \frac{1}{2}\right)$? Justify your answer. If you use a major theorem, state its name.

(c) i. Which of the following initial guesses for Newton's Method will fail to yield an approximation of a solution of $g(x) = 0$? (Circle each that applies. No justification is required.)

$$x = -1/\sqrt{3} \qquad x = 0 \qquad x = 1/\sqrt{3} \qquad x = 1.5$$

ii. Which of the following initial guesses for Newton's Method will most quickly converge to a solution of $g(x) = 0$? (Circle one. No justification is required.)

$$x = -1/\sqrt{3} \qquad x = 0 \qquad x = 1/\sqrt{3} \qquad x = 1.5$$

4. (16 pts) Using the grid below, sketch the graph of a **single function**, $y = f(x)$ with each of the following characteristics. (Sketch dashed lines to indicate any asymptotes that are present. The concavity of your graph should be clear.)

f is continuous on its domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$, $\quad f(-x) = f(x)$ for all x in the domain,

$f(0) = 0$, $\quad f'(x) > 0$ for $x < -3$, $0 < x < 2$, and $2 < x < 3$,

$f''(x) > 0$ for $-2 < x < 2$, $\quad f''(x) < 0$ for $x > 2$

$\lim_{x\to\infty} [f(x) - (-(1/2)x - 1)] = 0$, $\quad f(3) = -3$

$\lim_{x\to 2^-} f(x) = \infty$, $\quad \lim_{x\to 2^+} f(x) = -\infty$

5. (12 points) A rectangular box with an open top and square base is to have a surface area of 1200 square centimeters. Find the dimensions of such a box with the greatest volume. (Include the correct units in your final answer and be sure to justify that you have found the absolute maximum.)