# APPM 1340
# Exam 3
# Fall 2024

| **Name** | |
|---|---|
| **Instructor** Richard McNamara | **Section** 150 |

This exam is worth 100 points and has **4 problems**.

**Make sure all of your work is written in the blank spaces provided.** If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

**Show all work and simplify your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

**End-of-Exam Procedure**

1. Go to the designated area to scan and upload your exam to Gradescope.
2. Verify that your exam has been correctly uploaded and all problems have been labeled.
3. Hand the physical copy of your exam to a proctor.
4. Have a proctor swipe your BuffOne card.

1. (26 pts) Parts (a), (b), and (c) are not related.

   (a) Evaluate $\dfrac{d}{dx}\left[\cos^3(2x + x^{-3})\right]$.

   (b) Evaluate $\dfrac{d}{dx}\left[(x+1)^4(x+2)^5\right]$.

   Express your answer as a fully simplified product of three factors.

   (*Hint:* Identify common factors when simplifying.)

(c) Evaluate $\dfrac{d}{dx}\left[\tan x + \csc x\right]$ by rewriting both $\tan x$ and $\csc x$ in terms of $\sin x$ and $\cos x$ (as applicable), then applying the quotient rule to each quotient in the resulting expression.

Express your final answer in a form that does not include any $\sin x$ or $\cos x$ terms.

2. (22 pts) Parts (a) and (b) are not related.

(a) Find the equations of the tangent and normal lines to the curve $x^2 + xy + \sqrt{y} = 7x$ at the point $(1, 4)$.

(b) Find all values of $x$ at which the curve $y = 3x^{7/3} - 6x^{4/3} + 1$ has a horizontal tangent line.

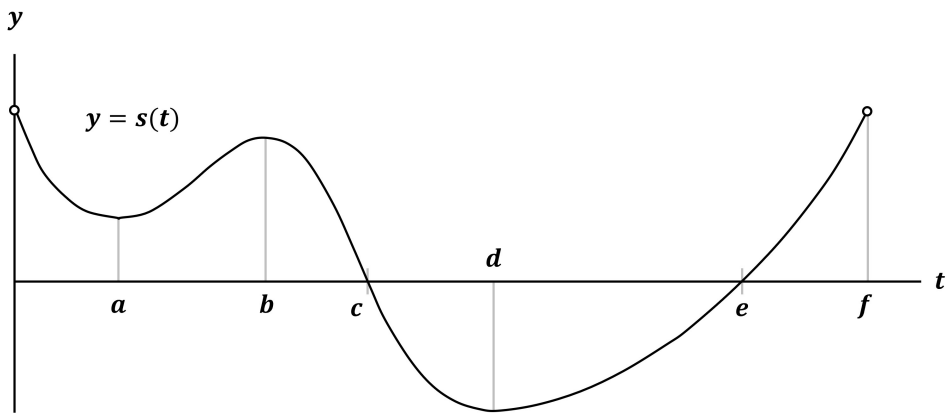
3. (26 pts) Parts (a) and (b) are not related.

   (a) The position function of a particle P is given by $s(t) = \cos t - \cos^2 t$, where $s$ is in meters, $t$ is in seconds, and $0 \leq t \leq \pi$.

      i. Identify particle P's acceleration function, $a(t)$, including the correct unit of measurement. You do **not** need to algebraically simplify your final answer.

      ii. Find the distance traveled by particle P between $t = 0$ and $t = \pi$ seconds, including the correct unit of measurement. Fully simplify your final answer.

(b) A graph of the position function for a particle Q for time $t$ on the open interval $(0, f)$ is provided below.




Answer the following questions, using $t$ values $0$, $a$, $b$, $c$, $d$, $e$, and $f$, as appropriate.

i. On what time interval(s), if any, is the velocity of particle Q positive? Express your answer using interval notation. You do not need to provide any justification for your answer.

ii. At what value(s) of $t$, if any, is particle Q at rest? You do not need to provide any justification for your answer.

iii. On time interval $(b, d)$, does particle Q's **speed** increase, decrease, or both? Explain your answer.

4. (26 pts) Parts (a) and (b) are not related.

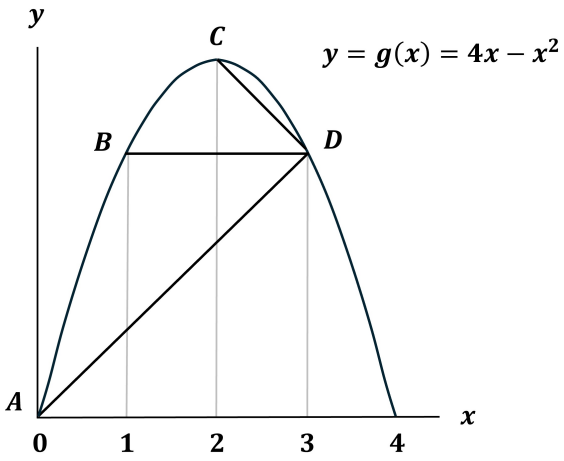
   (a) Consider the function $f(x)$, defined as follows:

$$f(x) = \begin{cases} x^2 & , \quad x < 1 \\ \\ 2x^2 - 2x + 1 & , \quad x \geq 1 \end{cases}$$

   i. Is $f(x)$ continuous at $x = 1$? Fully justify your answer using the limit definition of continuity.

   ii. Is $f(x)$ differentiable at $x = 1$? Fully justify your answer using the limit definition of $f'(1)$.

(b) The graph below depicts the curve $y = g(x) = 4x - x^2$. Let A, B, C, and D be points on the curve at $x = 0$, $x = 1$, $x = 2$, and $x = 3$, respectively.




i. Find the slope of the secant lines AD, BD, and CD. Show the details of your calculations and clearly indicate which secant line corresponds to each slope value.

ii. Use the limit definition of the derivative to determine the slope of the line that is tangent to the given curve at point D. You must evaluate an appropriate limit to earn full credit.

END OF EXAM

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.