INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from another person are not permitted while taking the exam. The exam is worth 100 points.

Potentially useful formulas:

Let u and w denote positive real numbers, then:

(a) $\log_b(u) = \dfrac{\log_a(u)}{\log_a(b)}$ for $a > 0$, $a \neq 1$.

(b) $A = \dfrac{1}{2}r^2\theta$

(c) $S = r\theta$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

iii. WRITE YOUR NAME ON THE NEXT PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND FOLLOW PROCTOR INSTRUCTIONS IN UPLOADING YOUR EXAM WITH SUPPORTING WORK TO GRADESCOPE. ONLY WORK THAT'S SUBMITTED TO GRADESCOPE WILL BE GRADED.

Name: _____

1. Determine the end behavior for the rational function: $R(x) = \dfrac{2x^4 + 3x^3 - 2x^2}{x^3 - 2x - 1}$. (5 pts)

2. Consider the following rational function: $r(x) = \dfrac{x^3 - 4x}{x^3 - 6x^2 + 8x}$. Answer the following: (12 pts)

(a) Find the x-coordinate of any hole(s) of $r(x)$. If there are none write NONE.

(b) Find the y-coordinate of any hole(s) you found in part (a). If there are none write NONE.

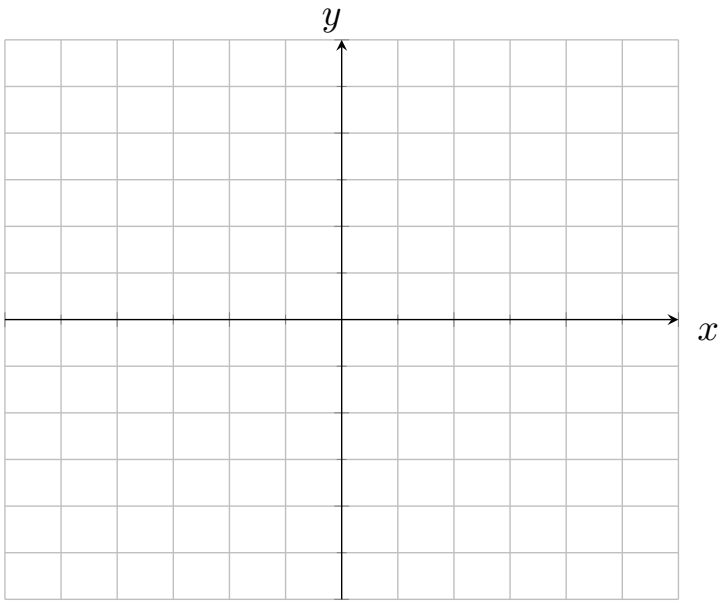
(c) Find all vertical asymptote(s) of $r(x)$. If there are none write NONE.

(d) Determine the end behavior of $r(x)$ and fill in the blanks: $r(x) \to$ ____ as $x \to -\infty$ and $r(x) \to$ ____ as $x \to \infty$.

(e) Find all x-intercept(s) of $r(x)$. If there are none write NONE.

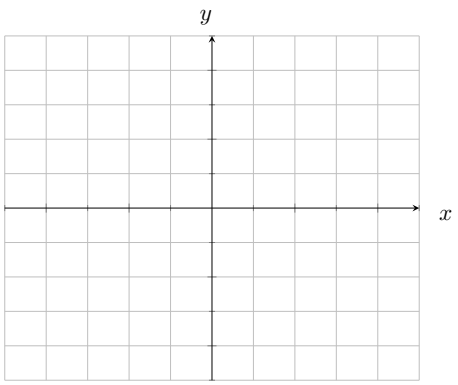
(f) Find the y-intercept. If there is none write NONE.

(g) Sketch the graph of $r(x)$ using parts (a)-(f). **Label** all intercept(s), hole(s), and asymptote(s) as relevant.

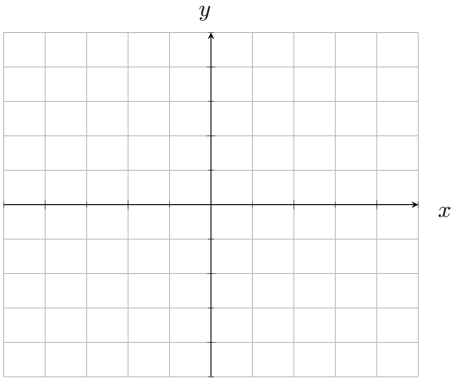


3. Sketch the following graphs: Be sure to **label** any asymptotes for each graph. (10 pts)

(a) $f(x) = -\log_2(x)$



(b) $g(x) = e^x - 2$



(c) Use the function given in part (b) of this question to determine the value: $g\left(\ln\left(\frac{1}{2}\right)\right)$.

(d) For $g(x) = e^x - 2$ graphed in part (b) fill in the blanks for the following:

$g(x) \to$ ____ as $x \to -\infty$ and $g(x) \to$ ____ as $x \to \infty$.

4. The following are unrelated.

(a) Simplify (rewrite without logs): $\ln(e) - \log_3(27) + 8^{2\log_8 5} + \log(1)$ (4 pts)

(b) Rewrite as a single logarithm: $5\log_2(x) - \frac{1}{2}(\log_2(y) + 7\log_2(x))$ (4 pts)

5. The half-life of Polonium-210 is 140 days. Suppose a sample of this substance has a mass of 130 mg. Use this information to help answer the following: (10 pts)

   (a) How many days until 65 mg remains?

   (b) Find a function $m(t) = m_\circ 2^{-t/h}$ that models the mass remaining after t days.

   (c) Find a function $m(t) = m_\circ e^{rt}$ that models the mass remaining after t days. Do not attempt to round (approximate) your value for r.

   (d) According to your model, how much of the mass remains after 30 days? Give the exact answer, do not attempt to round (approximate) your answer.

6. Solve the following equations for x. Do not attempt to round (approximate) your answers: (16 pts)

(a) $e^{0.7} = e^{x^2-1}$

(b) $\log 6 = \log x + \log(x-1)$

(c) $5^{2x} = 3^{7x+2}$

(d) $\log_2(x-4) = 3 + \log_2 5$

7. The area of a sector of a circle with a central angle of $\frac{7\pi}{5}$ radians is 6 m^2. Find the radius of the circle. (Give your answer in exact form, do not attempt to round (approximate).) (4 pts)

8. Suppose $\tan\theta = \frac{3}{7}$. Use this information to help answer the following: (8 pts)

   (a) Sketch a triangle that has acute angle θ.

   (b) Find $\sin\theta$.

   (c) Find $\cot\theta$.

9. If you know $\sin\theta < 0$ and $\cos\theta = \dfrac{1}{3}$ what quadrant must θ be in when graphed in standard position? No justification is required for this problem. (3 pts)

10. Find the exact value of the trigonometric function:

(a) $\sin(\pi)$ (3 pts)

(b) $\cos\left(\dfrac{5\pi}{3}\right)$ (3 pts)

(c) $\sin\left(-\dfrac{3\pi}{4}\right)$ (3 pts)

(d) $\csc(120^o)$ (3 pts)

(e) $\tan\left(\dfrac{11\pi}{6}\right)$ (3 pts)

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11. Evaluate (express your answer as a single fraction): (4 pts)

$$2\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$$

12. A 11 foot ladder leans against the vertical wall of a shed so that the upper end of the ladder just reaches the roof. If the ladder forms a 30° angle with the ground, how high is the roof? (5 pts)