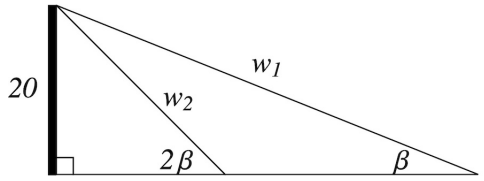
1. The following two problems are not related.

   (a) (10 pts) Find all values of θ in the interval $[0, 2\pi]$ that satisfy $\sin(2\theta) = \cos\theta$.

   (b) (14 pts) Two wires of different lengths are stretched tightly from the top of a 20-ft pole to the ground. The longer wire is w_1 feet long and forms an angle of β radians with the ground. The shorter wire is w_2 feet long and forms an angle of 2β radians with the ground. Given $\tan\beta = \frac{2}{5}$, find the following values and fully simplify your answers. (*Hint:* A double angle identity might be useful here.)

   i. w_1 ii. $\cos\beta$ iii. w_2



2. (18 pts) The function $f(x)$ is defined on the interval $[-5, 0) \cup (0, 5]$. Part of the formula for $f(x)$ is shown below:

$$f(x) = \begin{cases} -2x - 6 & \text{if } -5 \le x < -2 \\ -2 & \text{if } -2 \le x < 0 \\ \dots \end{cases}$$

   (a) For each expression, find the value if it exists. Otherwise write DNE. No justification is necessary.

   i. $(f \circ f)(-3)$ ii. $\lim_{x\to -2} f(x)$

   (b) Suppose $f(x)$ is odd. Write out the missing part of the formula for $f(x)$ on $(0, 5]$.

   (c) For each expression, find the value if it exists. Otherwise write DNE. No justification is necessary.

   i. $\lim_{x\to 5^-} f(x)$ ii. $\lim_{x\to 1/2} f\left(\frac{10}{x+2}\right)$

3. (20 pts) Evaluate the limit. If it does not exist, explain why. *(Reminder: You may not use L'Hopital's Rule in any solutions on this exam.)*

   (a) $\lim_{x\to 0} \frac{7x\sin x}{9x^2 + x\sin x}$ (b) $\lim_{x\to 0} x^{2/3}\cos\left(\frac{8}{x^2}\right)$

4. (18 pts) Consider the function

$$g(x) = \begin{cases} \dfrac{10 - \sqrt{x}}{100 - x} & \text{if } x > 100 \\ \dfrac{x}{c} & \text{if } x \le 100 \end{cases}$$

   where c is a nonzero constant.

(a) Evaluate $\lim_{x \to 100^+} g(x)$.

(b) Is there a value for c that will make $g(x)$ continuous at $x = 100$? If so, find c. If not, explain why there is no value. State the definition of continuity and use it to justify your answer.

5. (20 pts) Consider the function $h(x) = \dfrac{8x^2 - 48x}{x^2 - 36}$.

(a) Find the domain of $h(x)$. Express your answer in interval notation.

(b) Find the value of $\lim_{x \to 6} h(x)$ or explain why it does not exist.

(c) Shown at right is an illustration of $\lim_{x \to 6} h(x)$ using the precise definition of a limit. It shows that for $x \neq 6$, if x is in the interval $(5.7, 6.3)$, then $h(x)$ is in the interval $(L - 0.11, L + 0.11)$. Identify L, δ, and ϵ if they exist. No explanation is necessary.