1. (30pts) Let $f(x) = x\,e^{-x}$. You may use the fact that the kth derivative of f is

$$f^{(k)}(x) = (-1)^k e^{-x}(x-k), \qquad k \geq 1.$$

   (a) (15pts) Find the Maclaurin series for $f(x)$, written in sigma notation.

   (b) (5pts) Write $T_3(x)$, the third-order Maclaurin polynomial for f.

   (c) (10pts) Use Taylor's formula to find an error bound when $T_3(x)$ is used to approximate $f(x)$ on the interval $0 \leq x \leq 1$.

2. (20pts) Consider the parametric curve

$$x = t^2, \qquad y = t^3 - t, \qquad t \geq 0.$$

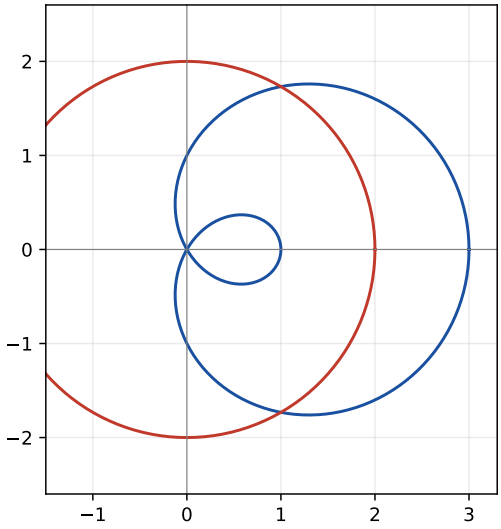
   (a) (10pts) Find an equation for the line tangent to the curve at the point where $t = 1$.

   (b) (10pts) The curve begins at the origin when $t = 0$. **Set up, but do not evaluate**, an integral that gives the length of the curve from the origin until the curve **next crosses the x-axis**.

3. (30pts) The following problems are not related.

   (a) (18pts) Consider the **cardioid** $r = 1 + \cos\theta$. Find all values of θ in $[0, 2\pi)$ at which the cardioid has a horizontal tangent line. If the slope is indeterminate, simply state the value of θ this occurs at.

   (b) (12pts) **Set up, but do not evaluate**, an integral that gives the area of the region that lies **inside** $r = 1 + 2\cos\theta$ and **outside** $r = 2$.



4. (45pts) Review soup! **Answer <u>THREE</u> of the following six problems.** Each is worth 15 points. **You may attempt as many as you like and your highest three will count towards your score**.

   (a) Evaluate $\displaystyle\int_0^{\pi/2} e^x \cos x\, dx$.

   (b) Solve the initial value problem for y as a function of x:

$$\frac{dy}{dx} = y\,\sec^4 x, \qquad y(0) = 1.$$

   (c) Let $\mathcal{R}$ be the region bounded by the line $y = 2x$ and the parabola $y = x^2$. Find the volume of the solid generated by rotating $\mathcal{R}$ about the line $x = 3$.

(d) Determine whether the improper integral converges or diverges. If it converges, find its value:

$$\int_1^\infty \frac{\ln x}{x^2}\,dx.$$

(e) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Fully justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{2n}}.$$

(f) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\,2^n}.$$