- This exam is worth 150 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”× 11” crib sheet with writing on both sides.

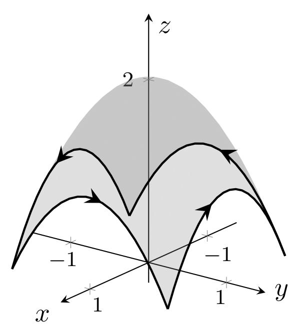
0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: ”I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/071726 (26 pts)] You are sitting under the tent shown in the figure below selling raffle tickets for guesses as to how many candies are in a glass jar. The tent is described by the function $z = 2 - x^2 - y^2$ over the region $-1 \le x \le 1, -1 \le y \le 1$. The wind in the area is given by the vector field $\mathbf{F}(x, y, z) = -yz\,\mathbf{i} + 2xy^2\,\mathbf{j} + 0\,\mathbf{k}$. A spider is crawling along the boundary of the tent in the direction shown and you are interested in how much work the spider does during its epic journey along the entire boundary.

   (a) (2 pts) What quadric surface describes the tent?

   (b) (2 pts) How many line integrals would be required to compute the work directly? Give a reason for your answer but do not evaluate any integrals.

   (c) (22 pts) State and use one of the Calculus III theorems to find the work without evaluating any line integrals. Zero points for using projections.



2. [2350/071726 (20 pts)] Use Green’s Theorem to evaluate $\oint_{\mathcal{C}} -y^3\,\mathrm{d}x + \left(x^3 + \sqrt{y^3+1}\right)\mathrm{d}y$. $\mathcal{C}$ is the closed counterclockwise path consisting of the bottom half of the unit circle and the portion of the x-axis with $-1 \le x \le 1$.

MORE PROBLEMS BELOW/ON REVERSE

3. [2350/071726 (23 pts)] You have a cool new "exponential" curtain covering a window in your apartment. It touches the floor ($xy-$plane) along the curve $y = e^{x/2}$ lying between $x = \ln 32$ and $x = \ln 60$. The top of the curtain reaches up to the surface $z = 3e^x$. All units are in feet. The fabric making up the curtain is sensitive to sunlight so you need to purchase some UV protection to spray on the side of the curtain facing the window. The UV protector comes in spray cans that each cover 100 square feet.

   (a) [20 pts] Use a line integral to determine how many full cans of the spray you must purchase to protect your curtain.

   (b) [3 pts] If you were provided a function $f(x, y, z)$ that gave the density (mass per unit area) of the material making up the curtain, what type of integral would you evaluate to compute the mass of the curtain?

4. [2350/071726 (28 pts)] The following problems are unrelated.

   (a) [8 pts] Let $\mathbf{V} = (x + 2y + az)\,\mathbf{i} + (bx - 3y - z)\,\mathbf{j} + (4x + cy + 2z)\,\mathbf{k}$. Find constants a, b and c so that $\int_{\mathcal{C}} \mathbf{V} \cdot d\mathbf{r}$ is path independent.

   (b) [20 pts] Find the work done by the conservative force field $\mathbf{F} = \left(2xy + z^3\right)\mathbf{i} + x^2\,\mathbf{j} + 3xz^2\,\mathbf{k}$ in moving an object along the path $\mathbf{r}(t) = \frac{4}{\pi}\left\langle \sin^{-1} t, \tan^{-1} t, \cos^{-1} t \right\rangle,\ -1 \le t \le 1$. Simplify your answer completely (*i.e.*, eliminate all trig functions)

5. [2350/071726 (53 pts)] You need to compute the downward flux of $\mathbf{F} = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{j} + \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{k}$ through that portion of the sphere of radius 2 centered at the origin lying below the plane $z = -1$ (not a closed surface). To accomplish this feat, let $\mathcal{E}$ be the solid region between the aforementioned sphere and plane. Its closed boundary, $\partial\mathcal{E}$, consists of the union of a disk, $\mathcal{S}_{\text{disk}}$, and the surface through which we seek the flux, $\mathcal{S}_{\text{sphere}}$, that is, $\partial\mathcal{E} = \mathcal{S}_{\text{disk}} \cup \mathcal{S}_{\text{sphere}}$.

   (a) [20 pts] Find the upward flux of $\mathbf{F}$ through $\mathcal{S}_{\text{disk}}$. No points for using projections.

   (b) [8 pts] Show that the divergence of $\mathbf{F}$ is $\dfrac{2}{\sqrt{x^2 + y^2 + z^2}}$.

   (c) [25 pts] Use Gauss' Divergence Theorem to find $\displaystyle\iint_{\mathcal{S}_{\text{sphere}}} \mathbf{F} \cdot d\mathbf{S}$. Zero credit for calculating the flux directly.