APPM 2360 **Exam 3** **Summer 2026**

- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”× 11” crib sheet with writing on one side.

0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: “I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/070626 (24 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a two-column table (letter - answer) completely separate from any work you do to arrive at the answer.

   (a) The general solution of the constant coefficient linear homogeneous differential equation whose characteristic equation is $\left(r^2-2r+4\right)^3=0$ is $y(t)=e^t\left[\left(c_1+c_2t+c_3t^2\right)\cos\sqrt{3}t+\left(c_4+c_5t+c_6t^2\right)\sin\sqrt{3}t\right]$

   (b) Let $x(t)=3e^{-2t}\cos t-2e^{-2t}\sin t+\sqrt{2}\cos(5t-\delta)$ be the equation of motion of an harmonic oscillator. The time-varying amplitude of the transient solution is $\sqrt{2}e^{-2t}$.

   (c) Let $x(t)$ be the displacement of an undamped harmonic oscillator subjected to a nonzero driving force, $f(t)$, that approaches zero as $t\to\infty$. Then $\lim\limits_{t\to\infty}x(t)=0$.

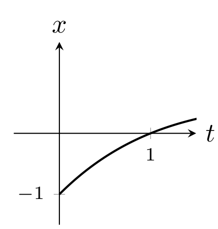
   (d) The differential equation $3\ddot{u}+2\cos u=5$ describes a conservative system.

   (e) The differential equation $y'''-y=1$ can be written as a system of differential equations in the form $\vec{\mathbf{x}}'=\mathbf{A}\vec{\mathbf{x}}$ where $\mathbf{A}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&1\end{bmatrix}$.

   (f) $\displaystyle\int_0^\infty e^{-st}\cos\left(\pi^2t\right)\,\mathrm{d}t=\frac{\pi^2}{s^2+\pi^2}$

   (g) Every nontrivial solution to a constant coefficient linear homogeneous differential equation whose characteristic roots are $1\pm3i$ exhibits oscillations with growing amplitude.

   (h) A portion of the equation of motion for an unforced mass/spring system is shown in the following figure. From the graph, the mass was released from rest 1 unit to the left of its equilibrium position.



2. [2360/070626 (23 pts)] Consider the linear operator $L(\vec{\mathbf{y}})=(1-\ln t)y''+\dfrac{y'}{t}-\dfrac{y}{t^2}$. Assume $t>e$.

   (a) (8 pts) Show that the solution space of $L(\vec{\mathbf{y}})=0$ equals span $\{t,\ln t\}$.

   (b) (15 pts) Find the general solution of $L(\vec{\mathbf{y}})=\dfrac{(1-\ln t)^2}{t}$.

MORE PROBLEMS AND LAPLACE TRANSFORM TABLE BELOW/ON REVERSE

3. [2360/070626 (22 pts)] You have decided to give your twin niece and nephew a sweet new toy for their upcoming birthday. It consists of a 1-kg bucket hooked to a spring which is then attached horizontally to a wall. The restoring/spring constant is $\frac{1}{4}$ N/m and the device is hooked up so that the damping force is equal to 4 times the instantaneous velocity of the bucket. A bag of 1-kg stones is included with the toy. Hint: There are no irrational numbers involved in this problem.

   (a) (15 pts) Upon seeing the toy, the kids put 6 stones into the 1-kg bucket, pull the bucket 1 meter to the right of the equilibrium position and impart a leftward velocity to it of one-half meter per second. How many times will the bucket pass through the equilibrium position? Justify your answer.

   (b) (4 pts) What is the smallest number of stones that need to be in the bucket so that the kids can have the excitement of watching the bucket oscillate? Justify your answer.

   (c) (3 pts) The kids really want a thrill and ask you to try to make the toy exhibit resonance. So you have them put 15 stones into the bucket and place the toy onto a table which provides a driving force to the toy given by $f(t) = \cos \frac{1}{8}t$. Will the kids get a thrill? Justify your answer.

4. [2360/070626 (15 pts)] Use the Method of Undetermined Coefficients to find the general solution of $\dfrac{\mathrm{d}^2 y}{\mathrm{d}t^2} - \dfrac{\mathrm{d}y}{\mathrm{d}t} = -40\cos^2 t$. Hint: $2\cos^2 t = 1 + \cos 2t$.

5. [2360/070626 (16 pts)] Evaluate the following.

   (a) (6 pts) $\mathscr{L}\left\{\left(t^2+3\right)^2 e^{-7t}\right\}$

   (b) (10 pts) $\mathscr{L}^{-1}\left\{\dfrac{5s-12}{s^2-6s+13}\right\}$

Short table of Laplace Transforms: $\mathscr{L}\{f(t)\} = F(s) \equiv \displaystyle\int_0^\infty e^{-st} f(t)\,\mathrm{d}t$

In this table, a, b, c are real numbers with $c \geq 0$, and $n = 0, 1, 2, 3, \ldots$

$$\mathscr{L}\left\{t^n e^{at}\right\} = \frac{n!}{(s-a)^{n+1}} \qquad \mathscr{L}\left\{e^{at}\cos bt\right\} = \frac{s-a}{(s-a)^2+b^2} \qquad \mathscr{L}\left\{e^{at}\sin bt\right\} = \frac{b}{(s-a)^2+b^2}$$

$$\mathscr{L}\{\cosh bt\} = \frac{s}{s^2-b^2} \qquad \mathscr{L}\{\sinh bt\} = \frac{b}{s^2-b^2}$$

$$\mathscr{L}\{t^n f(t)\} = (-1)^n \frac{\mathrm{d}^n F(s)}{\mathrm{d}s^n} \qquad \mathscr{L}\left\{e^{at} f(t)\right\} = F(s-a) \qquad \mathscr{L}\{\delta(t-c)\} = e^{-cs}$$

$$\mathscr{L}\{tf'(t)\} = -F(s) - s\frac{\mathrm{d}F(s)}{\mathrm{d}s} \qquad \mathscr{L}\{f(t-c)\,\text{step}(t-c)\} = e^{-cs}F(s) \qquad \mathscr{L}\{f(t)\,\text{step}(t-c)\} = e^{-cs}\mathscr{L}\{f(t+c)\}$$

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{(n-1)}(0)$$
