1. (30pts) Determine whether each of the following series converges or diverges. Justify your answers and name any test or theorem that you use.

   (a) (8pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2n^2+5}$

   (b) (10pts) $\displaystyle\sum_{n=1}^{\infty} n\, e^{-n^2}$

   (c) (10pts) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+1}{\sqrt{2n^5-n+4}}$

2. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n-2}$.

   (a) (12pts) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Fully justify your answer.

   (b) (8pts) How many terms of the series are needed to guarantee that the partial sum approximates the true sum with an error less than 0.01?

3. Consider the power series $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n\,(n+1)!}\,(x-2)^n$.

   (a) (10pts) Determine the radius of convergence for $f(x)$.

   (b) (10pts) Determine the interval of convergence for $f(x)$.

   (c) (6pts) For the following questions, you may use inequalities, interval notation, or list all the values (or say “there are no such x-values”).

      i. For which x values is this series absolutely convergent?

      ii. For which x-values is this series conditionally convergent?

      iii. For which x-values is this series divergent?

4. Let $f(x) = \sin^{-1}(x)$. Recall that $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$.

   (a) (6pts) Find the Maclaurin series for $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$.

   (b) (10pts) The binomial series you found in part (a) can be rewritten as

   $$\frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2}\, x^{2n}.$$

   Verify that the series you found in part (a) and this series agree for $n = 0, 1,$ and 2.

   (c) (8pts) Use the alternate formulation of the series from part (b) to find the Maclaurin series for $f(x) = \sin^{-1}(x)$ for $-1 < x < 1$ (Use $\sin^{-1}(0) = 0$.)

   (d) (2pts) State the radius of convergence of the series in part (c).