
INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources) are not permitted. **Give all answers in exact form.**

Potentially useful formulas:

$$\text{Volume of a cylinder } V = \pi r^2 h$$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

- i. DO NOT START UNTIL INSTRUCTED BY ME.
- ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE
- iii. WRITE YOUR NAME ON THE NEXT PAGE.
- iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY SCAN AND UPLOAD IT ON GRADESCOPE.

Name: _____

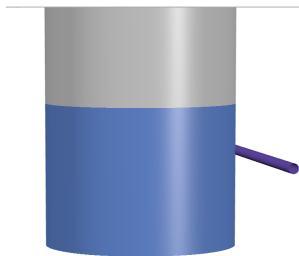
1. Consider the curve given implicitly by the equation

$$3x + xy^2 = 3 + x^2y$$

(a) (10 pts) Find $\frac{dy}{dx}$

(b) (10 pts) Find the equation of the tangent line to the curve at the point given by $y = 0$

2. (20 pts) Water is leaking from a cylindrical tank of radius 2 m and height 10 m at the rate of $10 \text{ m}^3/\text{s}$. At the same time, water is being pumped into the tank at the rate of $8 \text{ m}^3/\text{s}$. At what rate is the height of the water level decreasing?



3. (20 pts) Show that $f(x) = 2x - \sin x - 1$ has exactly one root in $[0, \pi]$.

(Hint: You will need the IVT as well as Rolle's theorem. For a complete solution, state the theorems clearly).

4. Consider the function $f(x) = \sin x - \frac{x}{\sqrt{2}}$ on the interval $[0, 2\pi)$

(a) (4 pts) Find the intervals where $f(x)$ is increasing and decreasing

(b) (4 pts) Find all the local extrema

(c) (4 pts) Find the intervals of concavity of f

(d) (4 pts) Find the point(s) of inflection

(e) (4 pts) Sketch a rough graph of the function. Make sure to label all the key points.

5. (20 pts) Find the absolute maximum and minimum values of

$$f(x) = 2 \cos x - \cos 2x, \quad x \in [0, 2\pi]$$

Hint: $\sin(2x) = 2 \sin x \cos x$