- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"$\times$ 11" crib sheet with writing on one side.

0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/062226 (15 pts)] Consider the linear system

$$x_1 + 2x_2 + x_3 = 4$$
$$x_1 + 2x_2 = 2$$
$$x_1 + x_2 + 3x_3 = -7$$

Use the inverse of the coefficient matrix to find the solution to the system. You must use elementary row operations to find the inverse. Hint: there are no fractions in the final answer.

2. [2360/062226 (22 pts)] The following parts (a) and (b) are not related. Fully justify all of your answers.

(a) (12 pts) Consider the set of functions in $\mathbb{P}_2$ given by $\left\{-5t+1, 4t^2-3t-2, 5t^2-\frac{13}{4}\right\}$.

i. (6 pts) Show that the Wronskian of the functions cannot be used to determine if the functions are linearly dependent or independent on the real line.

ii. (6 pts) Determine if the functions are linearly dependent or independent on the real line.

(b) (10 pts) Consider the set of vectors in $\mathbb{R}^3$ given by $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}2\\1\\-1\end{bmatrix}\right\}$.

i. (6 pts) Does the set of vectors span $\mathbb{R}^3$?

ii. (4 pts) Let $\mathbf{A}$ be the matrix formed using the vectors in the set as columns. Are there vectors $\vec{\mathbf{b}} \in \mathbb{R}^3$ that are not in Col $\mathbf{A}$? Justify your answer. Hint: This can be answered without doing any computations.

3. [2360/062226 (12 pts)] Determine if the following subsets $\mathbb{W}$ of the given vector space $\mathbb{V}$ are subspaces. Assume that the standard definitions of vector addition and scalar multiplication apply. Justify your answers.

(a) (6 pts) $\mathbb{V} = \mathbb{M}_{33}$; $\mathbb{W}$ is the set of matrices of the form $\begin{bmatrix} r & 0 & p \\ 0 & s & 0 \\ q & 0 & t \end{bmatrix}$ where p, q are real numbers and r, s, t are integers.

(b) (6 pts) $\mathbb{V} = \mathbb{R}^3$; $\mathbb{W}$ is the set of solutions to the equation $4x_1 - 3x_2 + 9x_3 = 0$.

MORE PROBLEMS BELOW/ON REVERSE

4. [2360/062226 (18 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a two-column table (letter - answer) completely separate from any work you do to arrive at the answer.

(a) For any matrix $\mathbf{A}$, $\left|\mathbf{A}^{\mathrm{T}}\mathbf{A}\right|$ and $\left|\mathbf{A}\mathbf{A}^{\mathrm{T}}\right|$ are defined.

(b) For invertible matrices $\mathbf{A}$ and $\mathbf{B}$, $\left|\mathbf{A}\mathbf{B}^{-1}\right| = |\mathbf{A}|/|\mathbf{B}|$.

(c) If square matrix $\mathbf{A}$ has 0 for an eigenvalue, then Cramer's Rule can be used to solve the system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

(d) Tr $\left[(2\mathbf{I})^3\right] = 24$ where $\mathbf{I}$ is the 3×3 identity matrix.

(e) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \in \text{span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

(f) Homogeneous systems of linear algebraic equations are never inconsistent.

5. [2360/062226 (10 pts)] Consider the augmented matrix $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$ derived from the linear system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$.

(a) (5 pts) If the matrix is in RREF, write the word YES. Otherwise, write NO and put the matrix in RREF.

(b) (5 pts) Find a basis for the solution space of the system. What is the dimension of the solution space?

6. [2360/062226 (23 pts)] The following parts (a) and (b) are not related.

(a) (11 pts) The characteristic polynomial of a certain matrix $\mathbf{B}$ is $p(\lambda) = 2\lambda^6 + 6\lambda^5 + 5\lambda^4$.

i. (2 pts) What is the order/size of the matrix $\mathbf{B}$?

ii. (6 pts) Find the eigenvalues of $\mathbf{B}$ and their algebraic multiplicities. Do not find the eigenvectors.

iii. (3 pts) Are the column vectors of $\mathbf{B}$ linearly dependent? Simply answer yes or no and justify your answer.

(b) (12 pts) Let $\mathbf{A} = \begin{bmatrix} -7 & 4 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 4 & -7 & -1 \\ 0 & 0 & 0 & -7 \end{bmatrix}$ and let $\mathbb{E}_{-7}$ denote the eigenspace associated with eigenvalue $\lambda = -7$.

i. (10 pts) Find a basis for $\mathbb{E}_{-7}$.

ii. (2 pts) What is the geometric multiplicity of the eigenvalue $\lambda = -7$?