

1. The following problems are unrelated.

(a) (6pts) Write out the form of the partial fraction decomposition (do NOT solve for coefficients) for

$$g(x) = \frac{x+1}{x^2(x-3)^3(x^2+4)^2}$$

(b) (12pts) Evaluate $\int_0^{\pi/4} x \sec^2 x \, dx$.

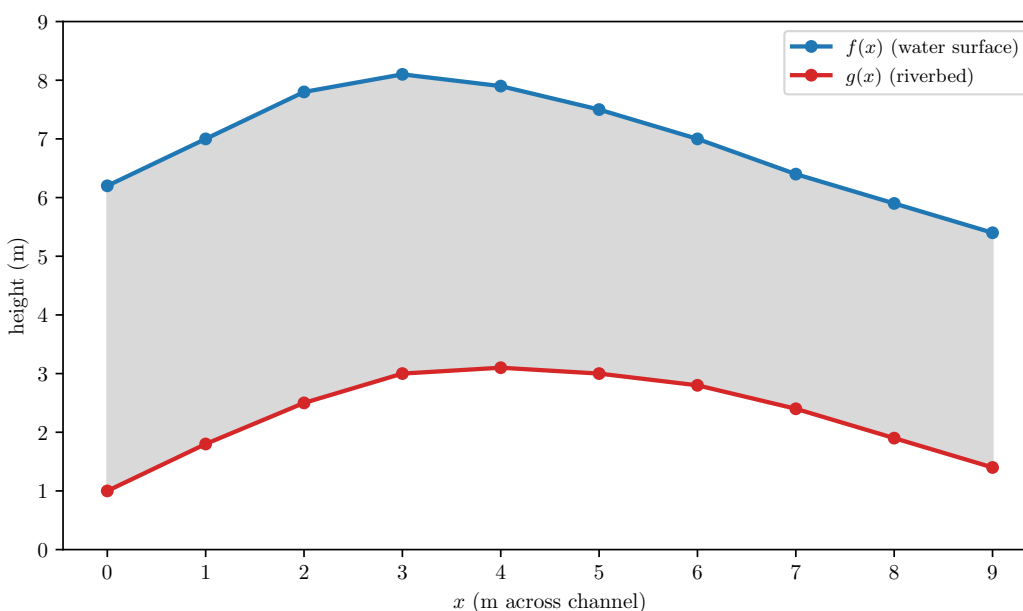
(c) (20pts) Evaluate $\int \frac{x^3}{\sqrt{x^2+4}} dx$

2. Determine whether the following integrals converge or diverge. If an integral converges you do not need to work out the value. If an integral diverges, justify why.

(a) (12pts) $\int_1^{\infty} \frac{2 + \cos x}{x^2} dx$

(b) (12pts) $\int_0^2 \frac{7x^2 + 12}{x^2(x^2+4)} dx$

3. A surveyor measures a cross-section of a river channel at the positions $x = 0, 1, 2, \dots, 9$ (in meters across the channel). The table records the water-surface height $f(x)$ and the riverbed height $g(x)$ (in meters above a fixed reference level):



x	0	1	2	3	4	5	6	7	8	9
$f(x)$	6.2	7.0	7.8	8.1	7.9	7.5	7.0	6.4	5.9	5.4
$g(x)$	1.0	1.8	2.5	3.0	3.1	3.0	2.8	2.4	1.9	1.4

(a) (10 pts) Using the trapezoidal rule with $n = 3$, approximate the cross-sectional area of the water in the channel (You do NOT need to simplify your expression, but you should give an expression which could be easily entered into a calculator).

(b) (10 pts) A smooth function $h(x)$ is fit to the data $f(x) - g(x)$ on $[0, 9]$, and it is known that $|h''(x)| \leq \frac{1}{9}$ on the interval. How large must n be to guarantee that the Trapezoidal Rule approximation has error less than 10^{-3} ? You should solve for n , but you do not need to simplify your answer.

4. Let R be the region bounded by $y^2 = x$ and $y = x - 2$.
- (a) (6pts) Sketch and shade R ; label intersection points.
 - (b) (12pts) Set up, but do not evaluate, an integral for the volume of the solid generated by rotating R about the line $x = 4$.