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INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources) are not permitted. **Give all answers in exact form.**

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**NOTE:** YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

- i. DO NOT START UNTIL INSTRUCTED BY ME.
- ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE
- iii. WRITE YOUR NAME ON THE NEXT PAGE.
- iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY SCAN AND UPLOAD IT ON GRADESCOPE.



Name: .....

1. The following questions are unrelated

(a) (3 pts) Simplify the expression

$$\pi \sin^2 \left( \frac{11\pi}{7} \right) + \pi \cos^2 \left( \frac{11\pi}{7} \right)$$

(b) (8 pts) Consider the function  $g$  such that  $g(x) = |x - 5|$

i. Is  $g$  odd, even, or neither? Please explain.

ii. State the domain and range of  $g$  in interval notation. No explanation required.

2. Evaluate the following limits

(a) (6 pts)  $\lim_{x \rightarrow 0} \frac{\sec x}{\csc x}$

(b) (6 pts)  $\lim_{x \rightarrow 0} \left( \frac{1}{2x} - \frac{1}{2x - x^2} \right)$

(c) (6 pts)  $\lim_{x \rightarrow -\infty} \frac{x + 2}{\sqrt{x^2 + 1}}$

3. Let  $f(x) = 2 + \frac{3 - 3x}{x^2 - 5x + 4}$

(a) (4 pts) Write the domain of  $f$  in interval notation.

(b) (6 pts) Does  $f$  have any removable discontinuity? Justify your answer using limits.

(c) (6 pts) Find the vertical asymptotes of  $f$ , if any. Justify using limits.

(d) (6 pts) Find the horizontal asymptotes of  $f$ , if any. Justify using limits.

4. The following questions are unrelated.

- (a) (8 pts) Explain whether the following function is continuous at  $x = 0$ , using the definition of continuity. Please show all your work clearly

$$f(x) = \begin{cases} \frac{x}{\sin(3x)} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

- (b) (8 pts) Show that the equation  $\cos x = x$  has at least one solution.

5. The following questions are unrelated.

- (a) (9 pts) Differentiate  $f(x) = \sqrt{4-x}$  using the limit definition of derivative.

- (b) (8 pts) For  $t \geq 0$  the position of a particle moving along the  $x$  axis is given by  $x(t) = \sin t - \cos t$ . What is the acceleration of the particle at the time when velocity is first equal to 0?

6. The following questions are unrelated.

(a) (7 pts) Differentiate  $f(x) = x \tan x + \frac{\sin x}{x}$

(b) (9 pts) Find the equation of the tangent line to the curve  $y = \sin(\cos x)$  at  $x = \frac{\pi}{2}$