APPM 3570/STAT 3100-001-002 Exam 2 Spring 2026

April 1, 2026

Instructor: Yu-Jui Huang

NAME: _____

SECTION: 001 *or* 002

Instructions:

1. A calculator and a 3×5 note card are permitted.
2. Your text and other books, cell phones, and other electronic devices are not permitted during the exam.
3. Justify your answers, show all work.
4. When you have completed the exam, go to the uploading area in the room and scan your exam and upload it to Gradescope.
5. Don't forget to scan any back pages you used for extra space!
6. Verify that everything has been uploaded correctly and the pages have been associated to the correct problems.
7. Turn in your hardcopy exam.

On my honor as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work.

Signature: _____ Date: _____

Duration: 90 minutes

Problem 1. (25 points) On average, 10 customers enter a store in a mall per hour. As the store manager, you arrive at the store at 11:00 a.m. Let N be the number of customers entering the store between 11:00 a.m. and 11:30 a.m. Let T be the amount of time (in hours) you wait until you see the first customer entering the store.

(a) Specify the distribution of N (i.e., the name of the distribution and its parameter(s)).

(b) Find the probability that there are at least 4 customers entering the store between 11:00 a.m. and 11:30 a.m.

(c) Specify the distribution of T (i.e., the name of the distribution and its parameter(s)).

(d) Find the probability that the first customer enters the shop after 11:10 a.m.

(e) Given that you have not yet seen the first customer at 11:10 a.m., what is the probability that the first customer will not show up until at least 11:20 a.m.?

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Problem 2. (25 points.) The height of 12-month old boys is normally distributed with a mean of 78 centimeters and a variance of 9. For the following questions, you may use the table on the last page of this exam.

(a) What is the probability that a 12-month old boy picked at random has a height between 75 and 79 centimeters?

(b) What is approximately the 75^{th} percentile for the height of 12-month old boys? (That is, for approximately which height are 75% of 12-month old boys shorter than this height?)

(c) Suppose that 64 12-month old boys are picked independently at random. Let N be the total number of boys among the 64 who have a height above the 75^{th} percentile.

(i) What is the distribution of N (i.e., the name of the distribution and its parameter(s))?

(ii) What is approximately the probability of $N = 16$? Use normal approximation to answer this question.

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Problem 3. (23 points) Let X be a random variable whose cumulative distribution function (cdf) is given by

$$F(x) = \begin{cases} \frac{1}{4(1+x^2)}, & x < 0; \\ \frac{x+1}{x+4}, & x \geq 0. \end{cases}$$

(a) Is X discrete or continuous? Justify your answer with rigorous mathematical arguments.

(b) If X is discrete, determine its probability mass function (pmf). If X is continuous, determine its probability density function (pdf).

(c) Find the probability that $-1 \leq X \leq 2$.

(d) Depending on your answer in (b), determine the pmf (or pdf) of the random variable $Y = 2X - 1$.

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Problem 4. (27 points.) Let (X, Y) be a random vector in $\mathbb{R}^2$ uniformly distributed on the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$.

(a) Find the joint probability density function (pdf) of X and Y.

(b) Find the marginal pdf of X and the marginal pdf of Y.

(c) Are X and Y independent? Justify your answer with rigorous mathematical arguments.

(d) Find the probability that $X \geq Y$.

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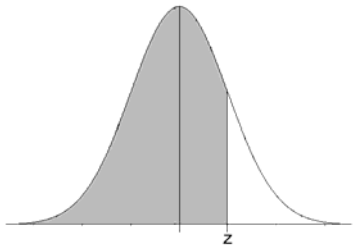
Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998