1. (25 points) Problems (a) and (b) are unrelated.

   (a) Consider the quadric surface given by
   $$9x^2 - y^2 - 2y + 3z^2 - 12z + 2 = 0.$$
      i. Determine the standard equation of the surface.
      ii. Identify the type of surface.

   (b) Consider plane A described by $x + 2y = 1$, and plane B described by $2z - x = 2$. Find the equation of the plane that is orthogonal to planes A and B, and contains the point $(1, 1, 1)$. Your final answer should be in the form $ax + by + cz = d$.

2. (25 points) Suppose D is the region in the xy-plane that is between the circles of radii 1 and 3 centered at the origin in the lower half plane, $y \leq 0$. Let $f(x, y) = \dfrac{1}{x^2 + y^2}$.

   (a) Sketch the region D on a clearly labeled xy-plane. Be sure to label all axes and intersection points on your sketch, and to shade in the region.

   (b) We will consider the integral $\iint_D f(x, y)\, dA$ by first setting it up two different ways, and then integrating just one of these two.
      i. Set up **but do not evaluate** the integral (or sum of integrals) $\iint_D f(x, y)\, dA$ using rectangular coordinates.
      ii. Set up **but do not evaluate** the integral (or sum of integrals) $\iint_D f(x, y)\, dA$ using polar coordinates.
      iii. Evaluate the expression you obtained in (i) **or** the expression you obtained in (ii). (If you evaluate both, clearly indicate which one you want to be graded.)

3. (25 points) Consider the force given by
   $$\mathbf{F}(x, y, z) = \langle 2x, e^z, ye^z \rangle.$$

   (a) Determine if the force, $\mathbf{F}$, is irrotational throughout $\mathbb{R}^3$.

   (b) Determine if the force, $\mathbf{F}$, is incompressible throughout $\mathbb{R}^3$.

   (c) Find the work done by $\mathbf{F}$ in moving an object from $(1, 1, 1)$ to $(2, -1, \ln 3)$. *If you use a Vector Calculus (Chapter 13) theorem, state its name and justify why it is applicable.*

4. (25 points) Consider the vector field
   $$\mathbf{F}(x, y, z) = \langle x^3, y^3, z^3 \rangle$$
   and the surface S that is the boundary of the region in space given by
   $$x^2 + y^2 \leq 1, \quad 0 \leq z \leq 2.$$
   Assume S has *outward orientation*.

   (a) Use Gauss' Divergence theorem to calculate the flux of $\mathbf{F}$ through S.

(b) Let S_1 denote the surface that is the side (cylinder) *and* the bottom disk ($z = 0$) of S, but *not* the top disk given by $z = 2$. Use your answer from (a) along with **only one additional surface integral** to calculate the flux of $\mathbf{F}$ through S_1.

5. (25 points) Let

$$\mathbf{F}(x, y, z) = \langle xy,\ xz,\ x^2 \rangle.$$

Let S be the surface given by

$$z = \frac{1}{2}\left(x^2 + y^2\right), \qquad 0 \le z \le 8.$$

Assume the *surface is oriented upward.*

(a) Set up and simplify, **but do NOT evaluate**, an integral that equals the flux of $\nabla \times \mathbf{F}$ through S. Your answer should be an appropriate integral in terms of an appropriate surface parameterization with a simplified integrand and clear limits of integration that would be ready to integrate (but again, you don't have to actually evaluate the integral).

(b) Use one of our Vector Calculus theorems (Chapter 13) to compute the flux of $\nabla \times \mathbf{F}$ through S. *State the name of the theorem that you use.* (Hint: You should write numerical quantities in a factored form to simplify your evaluation.)

6. (25 points) An astronaut is on the surface of a planet given by $x^2 + y^2 + 4z^2 = 9$. The temperature in this region of space is given by $T(x, y, z) = 2xy + 4z$.

(a) What quadric surface is the surface of the planet?

(b) Find the hottest point(s) and coldest point(s) on the surface of the planet, and the temperatures at these points. (Clearly indicate at which points the temperature is the hottest and at which points the temperature is the coldest.)

(c) Our astronaut is equipped with a drill and wants to find the hottest point(s) and coldest point(s) on the entire planet, including its interior. Would you advise the astronaut to stay on the surface of the planet, or to drill to a point inside the planet? Explain your reasoning.