1. The following are unrelated. Leave answers without negative exponents when relevant. (14 pts)

   (a) Add/subtract as indicated: $-\frac{1}{4}+\frac{7}{6}-\frac{\frac{5}{6}}{2}+\sqrt{45}$

   Solution:

$$-\frac{1}{4}+\frac{7}{6}-\frac{\frac{5}{6}}{2}+\sqrt{45} = -\frac{1}{4}+\frac{7}{6}-\frac{5}{12}+3\sqrt{5} \tag{1}$$
$$= -\frac{3}{12}+\frac{14}{12}-\frac{5}{12}+3\sqrt{5} \tag{2}$$
$$= \frac{6}{12}+3\sqrt{5} \tag{3}$$
$$= \boxed{\frac{1}{2}+3\sqrt{5}} \tag{4}$$

   (b) i. Express the quantity without using absolute value if we know $x<1$: $|x-1|$

   Solution:

   $x<1 \qquad \Rightarrow \qquad x-1<0$

$$|x-1| = \boxed{-(x-1)} \tag{5}$$
$$= \text{or } \boxed{1-x} \tag{6}$$

   ii. If we know $x<1$, perform the subtraction: $|x-1|-\frac{5}{2}x$.

   Solution:

$$|x-1|-\frac{5}{2}x = -(x-1)-\frac{5}{2}x \tag{7}$$
$$= 1-x-\frac{5}{2}x \tag{8}$$
$$= 1-\frac{2}{2}x-\frac{5}{2}x \tag{9}$$
$$= \boxed{1-\frac{7}{2}x} \tag{10}$$

   (c) Perform the indicated operations: $\left(-x^5\right)^2\left(\frac{x^{-1}}{x^5}\right)-\left(x^2-x\right)^2$

   Solution:

$$(-x^5)^2\left(\frac{x^{-1}}{x^5}\right)-(x^2-x)^2 = x^{10}\left(x^{-1-5}\right)-(x^2-x)^2 \tag{11}$$
$$= x^{10}\left(x^{-6}\right)-(x^2-x)^2 \tag{12}$$
$$= x^4-(x^2-x)^2 \tag{13}$$
$$= x^4-\left(x^4-2x^3+x^2\right) \tag{14}$$
$$= x^4-x^4+2x^3-x^2 \tag{15}$$
$$= \boxed{2x^3-x^2} \tag{16}$$

2. The following are unrelated. Simplify answers and leave without negative exponents. (9 pts)

(a) Perform the indicated operations: $\dfrac{x^2-x-2}{x^3-16x}\cdot\dfrac{x-4}{x^3-8}$

Solution:

$$\frac{x^2-x-2}{x^3-16x}\cdot\frac{x-4}{x^3-8}=\frac{x^2-x-2}{x^3-16x}\cdot\frac{x-4}{x^3-2^3} \tag{17}$$
$$=\frac{(x-2)(x+1)}{x(x^2-16)}\cdot\frac{x-4}{(x-2)(x^2+2x+4)} \tag{18}$$
$$=\frac{\cancel{(x-2)}(x+1)}{x\cancel{(x-4)}(x+4)}\cdot\frac{\cancel{(x-4)}}{\cancel{(x-2)}(x^2+2x+4)} \tag{19}$$
$$=\boxed{\frac{x+1}{x(x+4)(x^2+2x+4)}} \tag{20}$$

(b) Perform the indicated operations: $\sqrt{x}\left(\sqrt{x}-x^{2/3}\right)+\dfrac{1}{\left(x^{-1/9}\right)^3}$

Solution:

$$\sqrt{x}\left(\sqrt{x}-x^{2/3}\right)+\frac{1}{\left(x^{-1/9}\right)^3}=x^{1/2}\left(x^{1/2}-x^{2/3}\right)+\frac{1}{x^{-1/3}} \tag{21}$$
$$=x^{1/2+1/2}-x^{1/2+2/3}+x^{1/3} \tag{22}$$
$$=x-x^{7/6}+x^{1/3} \tag{23}$$
$$=\boxed{x-x^{7/6}+x^{1/3}} \tag{24}$$

3. Simplify (write answer without negative exponents): $\dfrac{\frac{1}{x}+1}{\frac{5}{x^2}+\frac{5}{x^3}}$ (4 pts)

Solution:

$$\frac{\frac{1}{x}+1}{\frac{5}{x^2}+\frac{5}{x^3}}=\frac{\frac{1}{x}+\frac{x}{x}}{\frac{5x}{x^3}+\frac{5}{x^3}} \tag{25}$$
$$=\frac{\frac{x+1}{x}}{\frac{5x+5}{x^3}} \tag{26}$$
$$=\frac{x+1}{x}\cdot\frac{x^3}{5x+5} \tag{27}$$
$$=\frac{x+1}{x}\cdot\frac{x^3}{5(x+1)} \tag{28}$$
$$=\frac{\cancel{x+1}}{x}\cdot\frac{x^3}{5\cancel{(x+1)}} \tag{29}$$
$$=\boxed{\frac{x^2}{5}} \tag{30}$$

4. Simplify: $\log_4(64)-\log_5\left(5^3\right)+\log_4(32)-\log_4(2)+\log(1)$ (Your answer should have no logarithms) (4 pts)

Solution:

$$\log_4(64)-\log_5(5^3)+\log_4(32)+\log(1)=3-\log_5(5^3)+\log_4\left(\frac{32}{2}\right)+\log(1) \tag{31}$$
$$=3-3+\log_4(16)+0 \tag{32}$$
$$=3-3+2+0 \tag{33}$$
$$=\boxed{2} \tag{34}$$

5. Find all real and complex solutions: $x^3 - x^2 + 4x = 0$. Give complex solutions in $a + bi$ form. (4 pts)

Solution:

$$x^3 - x^2 + 4x = 0 \tag{35}$$
$$x(x^2 - x + 4) = 0 \tag{36}$$

So $x = 0$ is one solution.

$$x^2 - x + 4 = 0 \tag{37}$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)} \tag{38}$$
$$= \frac{1 \pm \sqrt{1 - 16}}{2} \tag{39}$$
$$= \frac{1 \pm \sqrt{-15}}{2} \tag{40}$$
$$= \frac{1 \pm i\sqrt{15}}{2} \tag{41}$$

$$\boxed{x = 0,\ \frac{1}{2} + i\frac{\sqrt{15}}{2},\ \frac{1}{2} - i\frac{\sqrt{15}}{2}}$$

6. Solve the following equation for the indicated variable. If there are no solutions, write **no solutions**. (4 pts)

Solve for n: $\dfrac{1}{3n} = \dfrac{5}{n+1}$

Solution:

$$\frac{1}{3n} = \frac{5}{n+1} \tag{42}$$
$$n + 1 = 15n \tag{43}$$
$$1 = 14n \tag{44}$$

$$\boxed{n = \frac{1}{14}}$$

7. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (8 pts)

(a) Solve for r: $U - U_\circ = \dfrac{kqp}{r^2}$

Solution:

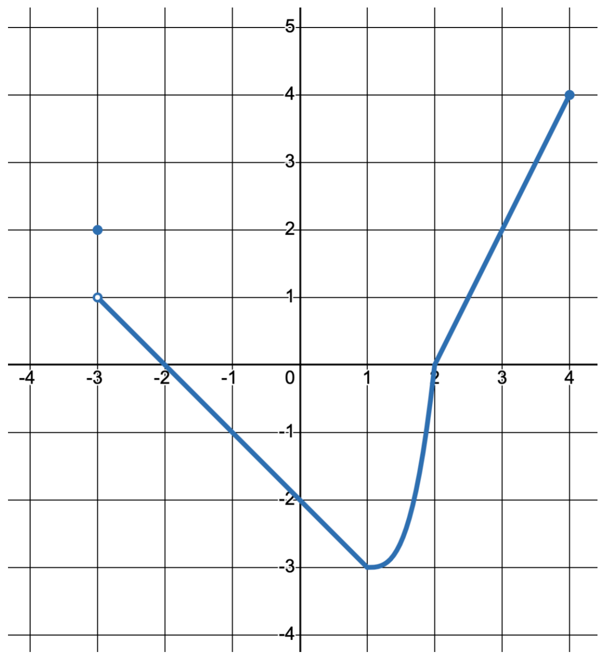
$$U - U_o = \frac{kqp}{r^2} \tag{45}$$
$$r^2(U - U_o) = kqp \tag{46}$$
$$r^2 = \frac{kqp}{U - U_o} \tag{47}$$
$$r = \boxed{\pm\sqrt{\frac{kqp}{U - U_o}}} \tag{48}$$

(b) Solve for x: $3 + 2\log_4(x) = 1$

Solution:

$$3 + 2\log_4(x) = 1 \tag{49}$$
$$2\log_4(x) = -2 \tag{50}$$
$$\log_4(x) = -1 \tag{51}$$
$$x = 4^{-1} \tag{52}$$
$$x = \boxed{\frac{1}{4}} \tag{53}$$

8. Use the graph of function $y = f(x)$ to answer the questions. (10 pts)



(a) Find the domain of $f(x)$.

Solution:

Domain: $[-3, 4]$

(b) Find the range of $f(x)$.

Solution:

Range: $[-3, 4]$

(c) Solve the inequality $f(x) < 0$.

Solution:

$f(x) < 0$: $x = (-2, 2)$

(d) Solve the equation $f(x) = 2$.

Solution:

$f(x) = 2$: $x = -3,\ 3$

(e) Solve the equation $(f \circ f)(2)$.

Solution:

$(f \circ f)(2) = f(f(2)) = f(0) = -2$

9. For the two points $P(-5,2)$ and $Q(1,7)$: (8 pts)

(a) Find the slope of the line through the two points.

Solution:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && (54)\\ &= \frac{7-2}{1-(-5)} && (55)\\ &= \boxed{\frac{5}{6}} && (56) \end{aligned}$$

(b) Find the equation of the line that passes through the points P and Q.

Solution:

$$\begin{aligned} y - y_1 &= m(x - x_1) && (57)\\ y - 2 &= \frac{5}{6}(x-(-5)) && (58)\\ y - 2 &= \frac{5}{6}(x+5) && (59)\\ y &= \frac{5}{6}x + \frac{25}{6} + 2 && (60) \end{aligned}$$

$$\boxed{y = \frac{5}{6}x + \frac{37}{6}}$$

(c) Find the equation of the circle centered at P that passes through Q.

Solution:

We know the circle is centered at $(-5,2)$. So we get:

$$(x+5)^2 + (y-2)^2 = r^2$$

The radius of the circle is found by using the distance formula:

$$\begin{aligned} r &= \sqrt{(1-(-5))^2 + (7-2)^2} && (61)\\ &= \sqrt{6^2+5^2} && (62)\\ &= \sqrt{36+25} && (63)\\ &= \sqrt{61} && (64) \end{aligned}$$

$$\boxed{(x+5)^2 + (y-2)^2 = 61}$$

10. Consider the functions: $m(x) = \sqrt{x-3}$, $r(x) = x^2 - 2$, and $q(x) = e^{3x}$. (12 pts)

(a) Find the domain of $m(x)$. Give your answer in interval notation.

Solution:

$$\begin{aligned} x - 3 &\geq 0 && (65)\\ x &\geq 3 && (66) \end{aligned}$$

$$\boxed{[3,\infty)}$$

(b) Find $(r \circ m)(x)$.

Solution:

$$\begin{aligned} (r \circ m)(x) &= r(m(x)) && (67)\\ &= r\left(\sqrt{x-3}\right) && (68)\\ &= \left(\sqrt{x-3}\right)^2 - 2 && (69)\\ &= x - 3 - 2 && (70)\\ &= \boxed{x-5} && (71) \end{aligned}$$

(c) Find the domain of $(r \circ m)(x)$. Give your answer in interval notation.

Solution:

$$\boxed{[3, \infty)}$$

(d) Find $q(\ln(4))$

Solution:

$$\begin{aligned} q(\ln(4)) &= e^{3\ln(4)} && (72)\\ &= e^{\ln(4^3)} && (73)\\ &= e^{\ln(64)} && (74)\\ &= \boxed{64} && (75) \end{aligned}$$

11. For $R(x) = \dfrac{x^2 - x - 6}{5x^2 - 5}$ answer the following (10 pts):

(a) Find the x-intercept(s). If there are none write NONE.

Solution:

The x-intercept(s) are found by setting $R(x) = 0$. We solve:

$$\begin{aligned} \frac{x^2 - x - 6}{5x^2 - 5} &= 0 && (76)\\ \frac{(x-3)(x+2)}{5(x-1)(x+1)} &= 0 && (77)\\ (5(x-1)(x+1)) \cdot \frac{x^2 - x - 6}{5(x-1)(x+1)} &= 0 \cdot (5(x-1)(x+1)) && (78)\\ x^2 - x - 6 &= 0 && (79)\\ (x+2)(x-3) &= 0 && (80) \end{aligned}$$

By using the multiplicative property of zero, the resulting x-intercepts are found when $x + 2 = 0$ and $x - 3 = 0$. The x-intercepts are thus $x = \boxed{-2}$ and $x = \boxed{3}$. Note that we factored the denominator to make sure the values are in the domain of $R(x)$.

(b) Find the y-intercept. If there is none write NONE.

Solution:

The y-intercept is found by setting $x = 0$. We plug in $x = 0$ and simplify:

$$\begin{aligned} R(0) &= \frac{0^2 - 0 - 6}{5 \cdot 0^2 - 5} && (81)\\ &= \frac{-6}{-5} && (82)\\ &= \boxed{\frac{6}{5}} && (83) \end{aligned}$$

(c) Find the horizontal or slant asymptote for $R(x)$. If there are none write NONE.

Solution:

The horizontal asymptote can be found by using the ratio of dominating terms in numerator and denominator. Specifically:

As $x \to \pm\infty$ then $R(x) = \dfrac{x^2 - x - 6}{5x^2 - 5} \to \dfrac{x^2}{5x^2} = \dfrac{1}{5}$

So the horizontal asymptote is $\boxed{y = \frac{1}{5}}$.

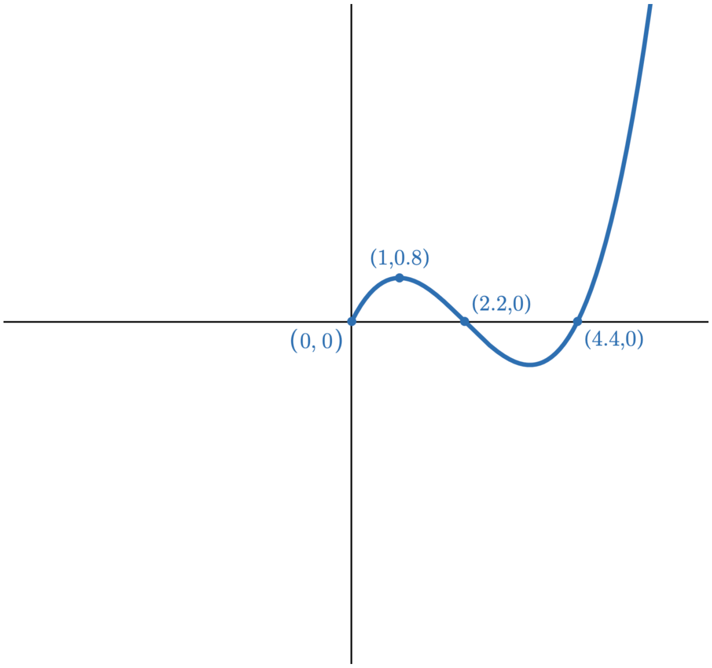
(d) Find all vertical asymptote for $R(x)$. If there are none write NONE.

Solution:

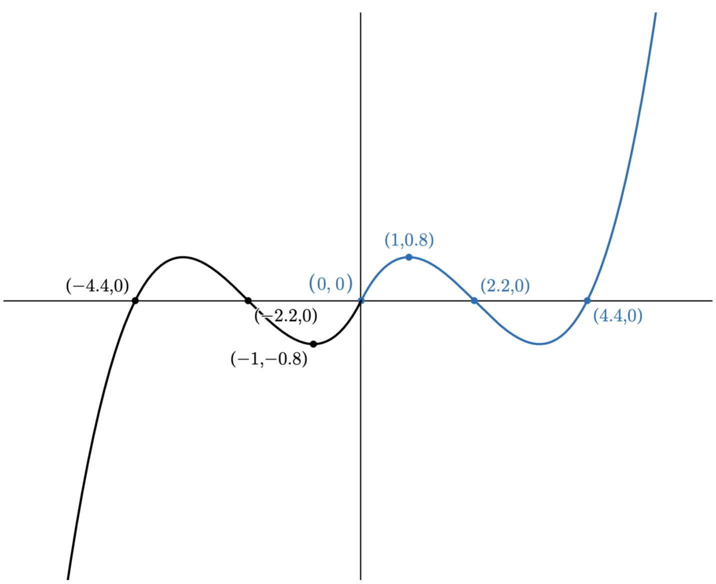
For the vertical asymptotes we factor numerator and denominator. $\dfrac{x^2-x-6}{5x^2-5} = \dfrac{(x-3)(x+2)}{5(x-1)(x+1)}$

Note that no factors cancel so there are no holes. This means the vertical asymptotes occur when the denominator is zero. Setting $5(x-1)(x+1)=0$ results in vertical asymptotes $\boxed{x=1}$ and $\boxed{x=-1}$.

12. (a) Below is the graph of a function $y=f(x)$ with domain $[0,\infty)$. Complete the graph (by drawing in the left side) so that the domain is $(-\infty,\infty)$ and is symmetric about the **origin**. (4 pts)



Solution:



(b) Find the y-value when $x=-1$. (2 pts)

Solution:

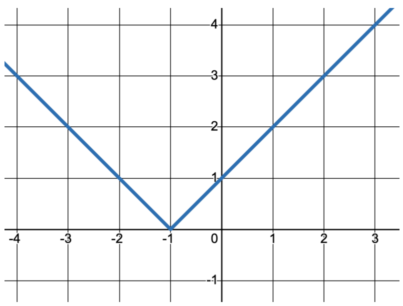
Using the definition of an odd function $f(-1) = -f(1) = \boxed{-0.8}$

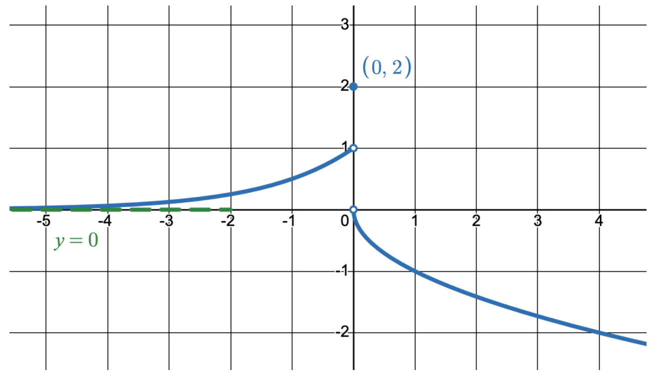
13. Sketch the graph of the following functions. **Label any asymptotes** as appropriate. (14 pts)

(a) $f(x) = |x+1|$.

(b) $r(x) = \begin{cases} 2^x & \text{if} \quad x < 0 \\ 2 & \text{if} \quad x = 0 \\ -\sqrt{x} & \text{if} \quad x > 0 \end{cases}$

Solution:

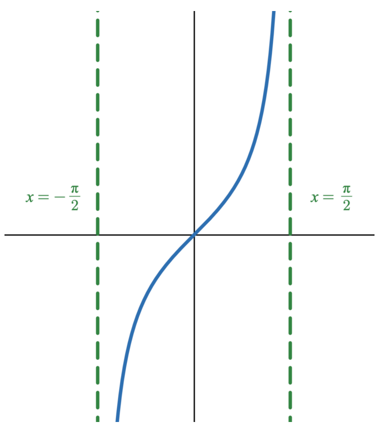


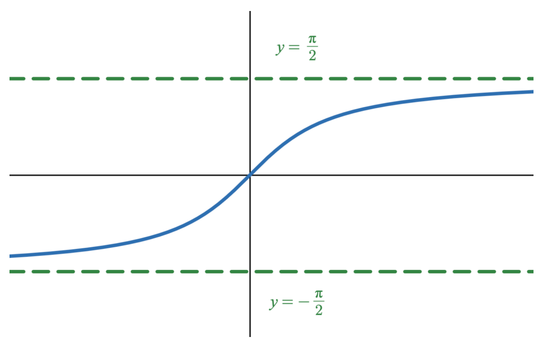


(c) $g(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(d) $h(x) = \tan^{-1}(x)$

Solution:





14. Find the exact value: (13 pts)

(a) $\sin\left(\frac{7\pi}{6}\right)$

(b) $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

Solution:

$$\sin\left(\frac{7\pi}{6}\right) = \boxed{-\frac{1}{2}}$$

$$\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{6}}$$

(c) $\cos\left(-\frac{3\pi}{2}\right)$

(d) $\arctan\left(\sqrt{3}\right)$

Solution:

$$\cos\left(-\frac{3\pi}{2}\right) = \boxed{0}$$

$$\arctan\left(\sqrt{3}\right) = \boxed{\frac{\pi}{3}}$$

15. Find the exact value: $\cos\left(\dfrac{5\pi}{8}\right)$ (5 pts)

Solution:

First we note that $\cos\left(\dfrac{5\pi}{8}\right)$ is negative since $\theta = \frac{5\pi}{8}$ lies in quadrant II. Thus when we use the half-angle formula $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ we will keep the negative: $\cos\left(\dfrac{\theta}{2}\right) = -\sqrt{\dfrac{1+\cos\theta}{2}}$.

To determine θ we can equate $\dfrac{\theta}{2} = \dfrac{5\pi}{8}$. Solving for θ we get the appropriate value to use in the formula: $\theta = \dfrac{5\pi}{4}$

$$\begin{aligned}
\cos\left(\frac{5\pi}{8}\right) &= \cos\left(\frac{\frac{5\pi}{4}}{2}\right) &(84)\\
&= -\sqrt{\frac{1+\cos\left(\frac{5\pi}{4}\right)}{2}} &(85)\\
&= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} &(86)\\
&= -\sqrt{\frac{2-\sqrt{2}}{4}} &(87)\\
&= -\frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}} &(88)\\
&= \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}} &(89)
\end{aligned}$$

16. Simplify the trigonometric expression: $\dfrac{\csc(\theta)\sin^2(\theta)-\cos(\theta)\cot(\theta)}{\csc(\theta)}$. (5 pts)

Solution:

$$\begin{aligned}
\frac{\csc(\theta)\sin^2(\theta)-\cos(\theta)\cot(\theta)}{\csc(\theta)} &= \frac{\frac{1}{\sin(\theta)}\sin^2(\theta)-\cos(\theta)\frac{\cos(\theta)}{\sin(\theta)}}{\frac{1}{\sin(\theta)}} &(90)\\
&= \left(\sin(\theta)-\frac{\cos^2(\theta)}{\sin(\theta)}\right)\cdot\sin(\theta) &(91)\\
&= \boxed{\sin^2(\theta)-\cos^2(\theta)} &(92)
\end{aligned}$$

17. Find all solutions to the following equations: (8 pts)

(a) $\sin(\theta) - \tan(\theta)\sin(\theta) = 0$

Solution:

$$\sin(\theta) - \tan(\theta)\sin(\theta) = 0 \tag{93}$$
$$\sin(\theta)\left(1 - \tan(\theta)\right) = 0 \tag{94}$$
$$\tag{95}$$

This results in equations $\sin(\theta) = 0$ and $1 - \tan(\theta) = 0$ or $\tan(\theta) = 1$. This results in solutions $\boxed{\theta = 0 + k2\pi}$, $\boxed{\theta = \pi + k2\pi}$, $\boxed{\theta = \frac{\pi}{4} + k2\pi}$, $\boxed{\theta = \frac{5\pi}{4} + k2\pi}$ where k is an integer. These answers can be written as: $\boxed{\theta = k\pi}$ and $\boxed{\theta = \frac{\pi}{4} + k\pi}$.

(b) $2\cos\left(\frac{\theta}{3}\right) = 1$

Solution:

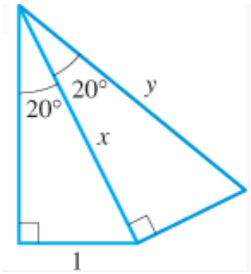
We can start by dividing both sides by 2 to isolate $\cos\left(\frac{\theta}{3}\right)$. Then we can let $u = \frac{\theta}{3}$.

$$2\cos\left(\frac{\theta}{3}\right) = 1 \tag{96}$$
$$\cos\left(\frac{\theta}{3}\right) = \frac{1}{2} \tag{97}$$
$$\cos(u) = \frac{1}{2} \tag{98}$$

$\cos(u) = \frac{1}{2}$ is solved when $u = \frac{\pi}{3} + k2\pi$ and $u = \frac{5\pi}{3} + k2\pi$ for k an integer.

Substituting $u = \frac{\theta}{3}$ back in we get $\frac{\theta}{3} = \frac{\pi}{3} + k2\pi$ and $\frac{\theta}{3} = \frac{5\pi}{3} + k2\pi$. Multiplying by 3 on both sides we get: $\theta = \boxed{\pi + k6\pi}$ and $\theta = \boxed{5\pi + k6\pi}$.

18. Solve for both x and y. Leave your answer in exact form, do not attempt to approximate your answer with a decimal value. (6 pts)



Solution:

Using SOHCAHTOA on the left triangle we get $\sin(20°) = \frac{1}{x}$. Multiplying both sides by x we get $x\sin(20°) = 1$ and dividing by $\sin(20°)$ we get $\boxed{x = \frac{1}{\sin(20°)}}$. Using SOHCAHTOA on the right triangle we get $\cos(20°) = \frac{x}{y}$. Plugging in for x we get $\cos(20°) = \frac{\frac{1}{\sin(20°)}}{y}$ which simplifies to $\cos(20°) = \frac{1}{y\sin(20°)}$ multiplying both sides by $y\sin(20°)$ we get $y\sin(20°)\cos(20°) = 1$ and solving for y we get $\boxed{y = \frac{1}{\cos(20°)\sin(20°)}}$.

19. For $f(x) = 2\sin\left(x - \frac{\pi}{3}\right)$ (7 pts)

(a) Identify the amplitude.

Solution: $\boxed{2}$

(b) Identify the period.

Solution: $\boxed{2\pi}$

(c) Identify the phase shift.

Solution: $\boxed{\frac{\pi}{3}}$

(d) Sketch one cycle of the graph of $f(x)$. Be sure to label at least two values on the x-axis and clearly identify the amplitude.