APPM 1235 **Final Exam** **Spring 2026**

INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no/insufficient justification may receive no/limited credit. **Not allowed:** Calculators, notes, phones, electronic earphones, electronic devices, other unauthorized devices, and getting help from (or giving help to) another person while taking the exam. The exam is worth 150 points.

Potentially useful formulas:

1. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

1. The following are unrelated. Leave answers without negative exponents when relevant. (14 pts)

(a) Add/subtract as indicated: $-\frac{1}{4}+\frac{7}{6}-\frac{\frac{5}{6}}{2}+\sqrt{45}$

(b) i. Express the quantity without using absolute value if we know $x<1$: $|x-1|$

ii. If we know $x<1$, perform the subtraction: $|x-1|-\frac{5}{2}x$.

(c) Perform the indicated operations: $\left(-x^5\right)^2\left(\frac{x^{-1}}{x^5}\right)-\left(x^2-x\right)^2$

2. The following are unrelated. Leave answers without negative exponents when relevant. (9 pts)

(a) Perform the indicated operations: $\dfrac{x^2-x-2}{x^3-16x} \cdot \dfrac{x-4}{x^3-8}$

(b) Perform the indicated operations: $\sqrt{x}\left(\sqrt{x}-x^{2/3}\right)+\dfrac{1}{\left(x^{-1/9}\right)^3}$

3. Simplify (write answer without negative exponents): $\dfrac{\frac{1}{x}+1}{\frac{5}{x^2}+\frac{5}{x^3}}$ (4 pts)

4. Simplify: $\log_4(64) - \log_5\left(5^3\right) + \log_4(32) - \log_4(2) + \log(1)$ (Your answer should have no logarithms) (4 pts)

5. Find all real and complex solutions: $x^3 - x^2 + 4x = 0$. Give complex solutions in $a + bi$ form. (4 pts)

6. Solve the following equation for the indicated variable. If there are no solutions, write **no solutions**. (4 pts)

   Solve for n: $\dfrac{1}{3n} = \dfrac{5}{n+1}$

7. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (8 pts)

(a) Solve for r: $U - U_\circ = \dfrac{kqp}{r^2}$

(b) Solve for x: $3 + 2\log_4(x) = 1$

8. Use the graph of function $y = f(x)$ to answer the questions. (10 pts)

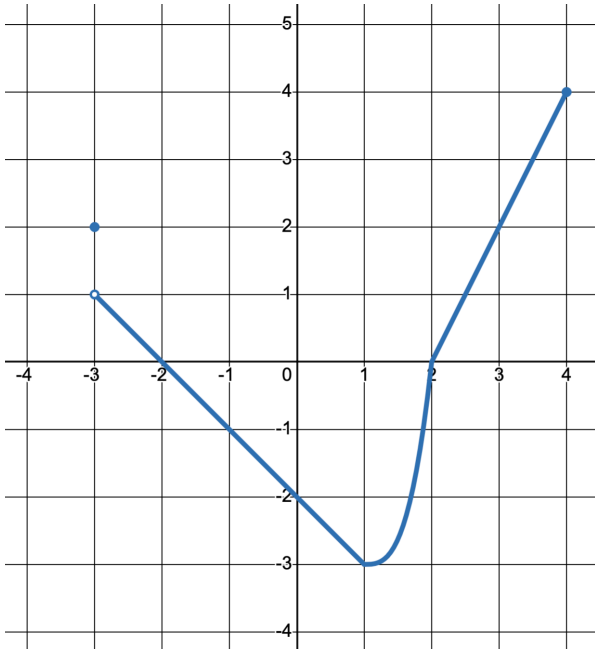
(a) Find the domain of $f(x)$.

(b) Find the range of $f(x)$.

(c) Solve the inequality $f(x) < 0$.

(d) Solve the equation $f(x) = 2$.

(e) Evaluate $(f \circ f)(2)$.



9. For the two points $P(-5, 2)$ and $Q(1, 7)$: (8 pts)

   (a) Find the slope of the line through the two points.

   (b) Find the equation of the line that passes through the points P and Q.

   (c) Find the equation of the circle centered at P that passes through Q.

10. Consider the functions: $m(x) = \sqrt{x-3}$, $r(x) = x^2 - 2$, and $q(x) = e^{3x}$. (12 pts)

   (a) Find the domain of $m(x)$. Give your answer in interval notation.

   (b) Find $(r \circ m)(x)$.

   (c) Find the domain of $(r \circ m)(x)$. Give your answer in interval notation.

   (d) Find $q(\ln(4))$

11. For $R(x) = \dfrac{x^2 - x - 6}{5x^2 - 5}$ answer the following (10 pts):

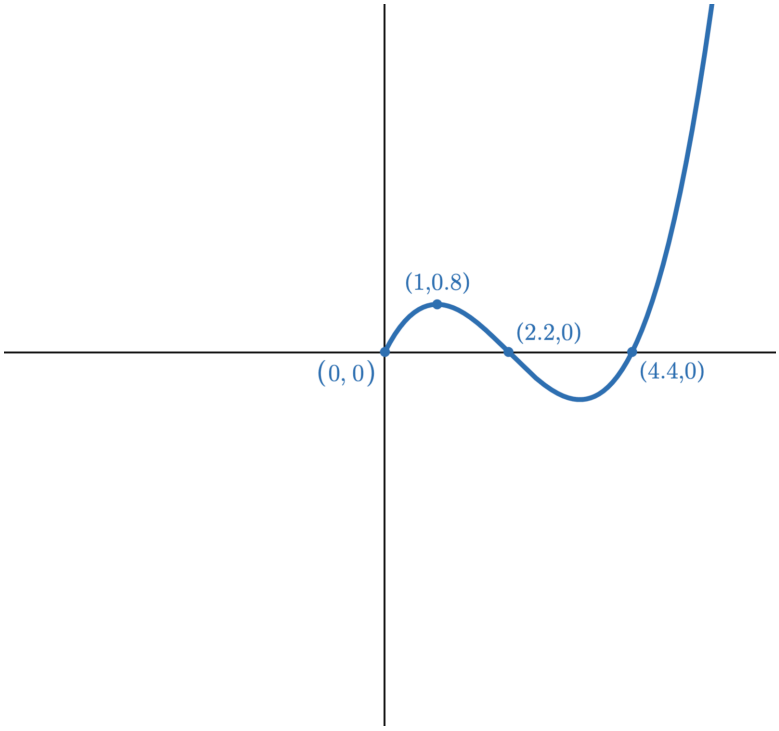
(a) Find the x-intercept(s). If there are none write NONE.

(b) Find the y-intercept. If there is none write NONE.

(c) Find the horizontal or slant asymptote for $R(x)$. If there isn't one write NONE.

(d) Find all vertical asymptote for $R(x)$. If there are none write NONE.

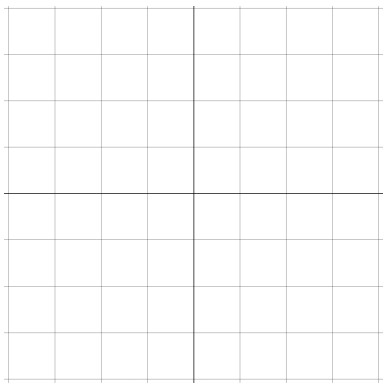
12. (a) Below is the graph of a function $y = f(x)$ with domain $[0, \infty)$. Complete the graph (by drawing in the left side) so that the domain is $(-\infty, \infty)$ and is symmetric about the **origin**. (4 pts)



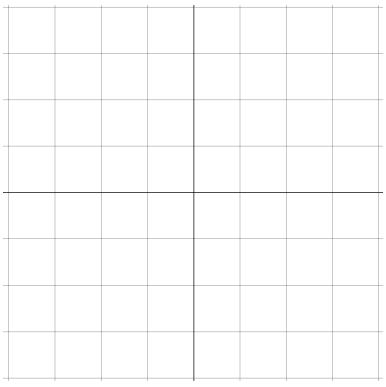
(b) Use part (a) to find the y-value when $x = -1$. (2 pts)

13. Sketch the graph of the following functions. **Label any asymptotes** as appropriate. (13 pts)

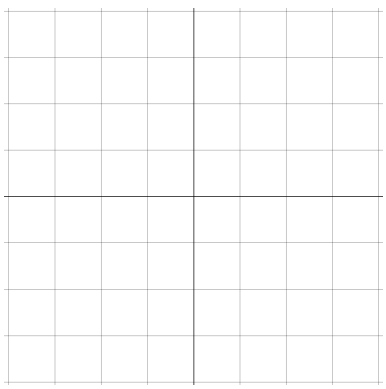
(a) $f(x) = |x + 1|$.



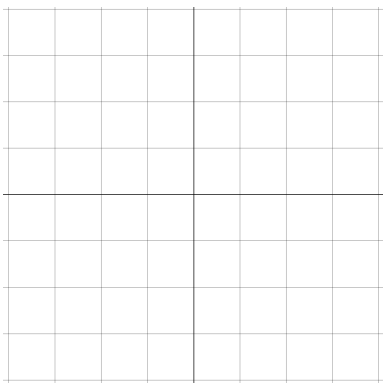
(b) $r(x) = \begin{cases} 2^x & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ -\sqrt{x} & \text{if } x > 0 \end{cases}$



(c) $g(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



(d) $h(x) = \tan^{-1}(x)$



14. Find the exact value: (13 pts)

(a) $\sin\left(\frac{7\pi}{6}\right)$

(b) $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

(c) $\cos\left(-\frac{3\pi}{2}\right)$

(d) $\arctan\left(\sqrt{3}\right)$

15. Find the exact value: $\cos\left(\frac{5\pi}{8}\right)$ (5 pts)

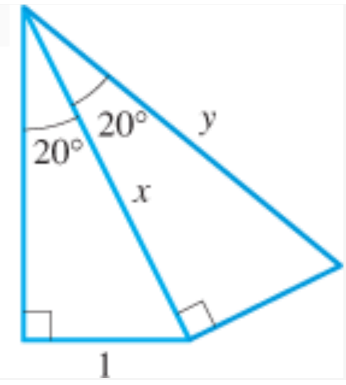
16. Simplify the trigonometric expression: $\frac{\csc(\theta)\sin^2(\theta) - \cos(\theta)\cot(\theta)}{\csc(\theta)}$. (5 pts)

17. Find all solutions to the following equations: (8 pts)

(a) $\sin(\theta) - \tan(\theta)\sin(\theta) = 0$

(b) $2\cos\left(\frac{\theta}{3}\right) = 1$

18. Solve for both x and y. Leave your answer in exact form, do not attempt to approximate your answer with a decimal value. (6 pts)



19. For $f(x) = 2\sin\left(x - \frac{\pi}{3}\right)$ (7 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $f(x)$. Be sure to label at least two values on the x-axis and clearly identify the amplitude.