1. (24 pts) Evaluate the following integrals:

   (a) $\displaystyle\int_{-\infty}^{0} xe^{x}\,dx$

   (b) $\displaystyle\int \sin^5\theta\cos^3\theta\,d\theta$

2. (24 pts) Determine whether the following quantities converge or diverge. Remember to state any tests or theorems you use, and check their conditions.

   (a) $\{a_n\}_{n=3}^{\infty}$, where $a_n = \dfrac{\sin(n^2+1)\cos(n)}{n-2}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+3)!}{\sqrt{n+1}\cdot 3^n}$

3. (20 pts) Consider the region in Quadrant 1 which lies between the curves

$$x = 2y - y^2,$$
$$x = y.$$

   (a) Sketch this region, and label any points of intersection.

   (b) Set up, but do not evaluate, an integral which gives the volume of the solid generated by revolving this region around $y = -1$.

4. (20 pts) Consider the integral $\displaystyle\int_0^1 \ln\left(\frac{1}{2}x+1\right)\,dx$. All parts of this problem refer to this integral.

   (a) Use the Maclaurin series for $\ln(x+1)$ to find a series representation for this integral (make sure to evaluate the bounds; your answer should not have an integral sign in it).

   (b) Use the Alternating Series Estimation theorem to determine the error bound if the first four nonzero terms from (a) are used to approximate the value of the integral.

5. (24 pts) Suppose a bird's position is given by the equations

$$x(t) = t^3 + t + 1$$
$$y(t) = \sqrt{t},$$

   for $0 \le t \le 10$, where $x(t)$ is the bird's horizontal position and $y(t)$ is the bird's height off the ground at time t.

   (a) At which (x, y) point does the bird start? When does the bird land on the ground again in the given time interval? Give a specific time, or say "never" with appropriate justification.

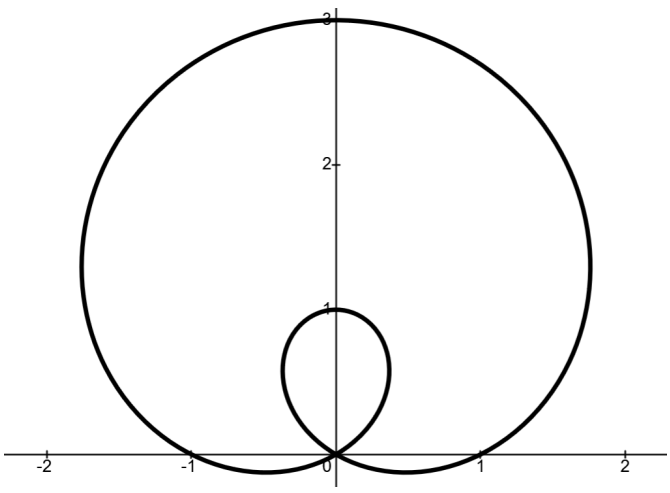
   (b) What is the slope of the tangent line to the bird's path at $t = 2$?

   (c) Set up, but do not evaluate, an integral which gives the distance the bird travels over this time interval. In other words, your integral should give the length of the path in the xy-plane.

6. (22 pts) Consider the polar curve given by

$$r = 1 + 2\sin\theta, \qquad \text{for } 0 \le \theta \le 2\pi,$$

   which is plotted here:



(a) Give an expression for the slope of a tangent line to this curve as a function of θ.

(b) Set up, but do not evaluate, an integral that gives the area of the inner loop.

7. (16 pts) Consider the conic section given by the equation

$$x^2 + 5y^2 + 10y + 4x = 1.$$

(a) Determine whether this conic section is a parabola, ellipse, or hyperbola.

(b) If you said **ellipse** or **hyperbola** for (a), determine the center of the conic section. If you said **parabola**, determine its vertex.