APPM 2360 **Final Exam** **Spring 2026**

- This exam is worth 150 points and has 8 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”$\times$ 11” crib sheet with writing on two sides.

---

0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: “I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/042726 (25 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a two-column table (letter - answer) completely separate from any work you do to arrive at the answer.

   (a) An undamped oscillator with circular frequency $\omega_0 = 3$ is forced by $10t\sin 3t$. The guess for the particular solution when using the method of undetermined coefficients is $x_p = (At+B)\sin 3t + (Ct+D)\cos 3t$.

   (b) If $\vec{\mathbf{b}} \in$ Col $\mathbf{A}$, where $\mathbf{A}$ has only positive eigenvalues, then there exists a unique vector $\vec{\mathbf{x}}$ such that $\vec{\mathbf{x}} - \mathbf{A}^{-1}\vec{\mathbf{b}} = \vec{\mathbf{0}}$.

   (c) If $\mathbf{A}$ is nonsingular, then $\left|\mathbf{A}\mathbf{A}^{\mathrm{T}}\mathbf{A}^{-1}\right| = |\mathbf{A}|$.

   (d) If $f_y(t_0, y_0)$ is not defined, then $y' = f(t,y),\ y(t_0) = y_0$ cannot have a unique solution.

   (e) Euler’s method cannot be used to approximate the solution of $y' = y^{-1},\ y(1) = 0$ even though the initial value problem possesses a unique solution.

   (f) The set, $\mathbb{W}$, of points in $\mathbb{R}^2$ lying on either the x-axis or y-axis is a subspace of $\mathbb{R}^2$.

   (g) The system $\begin{array}{l} x' = x^4 + y^2 + 1 \\ y' = x + y - 1 \end{array}$ possesses at least one equilibrium solution.

   (h) If $a > 0$, solutions to $y'' + \sqrt{a}y' + ay = 0$ all approach 0 as $t \to \infty$.

   (i) $\mathbb{S} =$ span $\{t, \cos t\}$ is the solution space of $(t\cos t - \sin t)\,y'' + (t\sin t)\,y' - (\sin t)\,y = 0$.

   (j) There exists a curve in the xy-plane such that if solutions to the differential equation $e^y\,(y'+1) = 2e^x$ cross this curve, they do so with a slope of 1.

2. [2360/042726 (17 pts)] Wildlife biologists have determined that the population of hummingbirds in a certain region is growing logistically. At the same time, these biologists have noted that a parasitic infection is killing the birds at a constant rate of 49 thousand birds per year. The differential equation governing this scenario is $p'(t) = (14-p)p - 49$. $p(t)$ gives the number of hummingbirds (in thousands) at time t (years).

   (a) (2 pts) Draw a properly labeled phase line for the equation and state the stability of all equilibrium solutions.

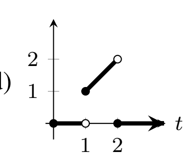
   (b) (3 pts) Are there initial hummingbird populations, p_0, that will tend to a nonzero steady state as t goes to infinity? If so, find those initial populations as well as the nonzero steady state which they will approach. If not, explain why not. Hint: you need not solve the differential equation to answer this.

   (c) (12 pts) If there are initially 6000 hummingbirds $[p(0) = 6]$ will the population sustain itself or will the birds go extinct in a finite amount of time? If the birds go extinct, when will that occur? If they don’t, explain why not.

3. [2360/042726 (15 pts)] Find the general solution of the linear system consisting of the three equations
$$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 &= -1 \\ 6x_2 - 18x_4 &= 12. \\ 3x_1 - 2x_3 &= 7 \end{aligned}$$
   Solve this system by finding the RREF of an appropriate matrix. Write your answer using the Nonhomogeneous Principle. In addition, find the solution space of the associated homogeneous system and state its dimension. Hints: create the appropriate matrix very carefully before starting; the correct answer does not contain fractions; zero credit for using a method other than the RREF.

4. [2360/042726 (16 pts)] For parts (a) and (b) perform the operations. For parts (c) and (d), write $f(t)$ using step functions.

   (a) $\mathscr{L}^{-1}\left\{\dfrac{e^{-2s}(s-1)}{(s-1)^2+b^2}\right\}$ (b) $\mathscr{L}\left\{(2-3t)\,\text{step}(t-1)\right\}$ (c) $f(t) = \begin{cases} 2 & t < 3 \\ -4 & 3 \le t < 4 \\ 1 & t \ge 4 \end{cases}$ (d)

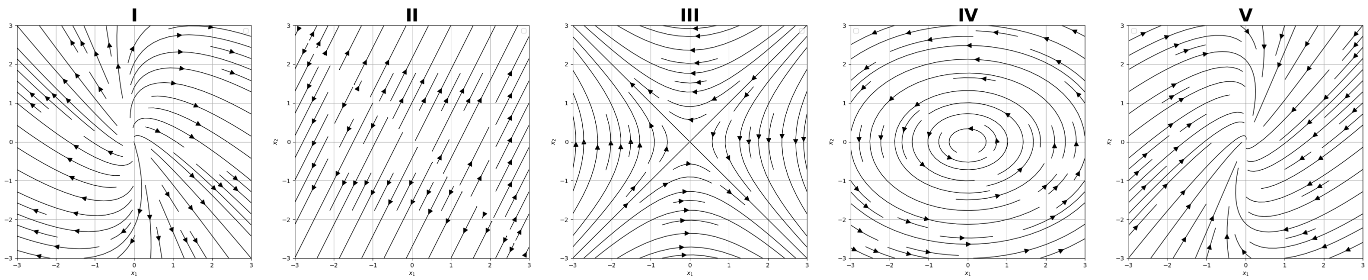


MORE PROBLEMS AND LAPLACE TRANSFORM TABLE BELOW/ON REVERSE

5. [2360/042726 (15 pts)] Sid Phillips' pop tarts just came out of the toaster having a temperature of 50 °C. His mom places them in a room where the temperature is 20 °C when the wall clock says noon. At exactly 12:05 PM, Buzz Lightyear sees the pop tarts, is frightened and blasts them with his laser, giving them an impulsive temperature change of 6°C. Assuming Newton's law of cooling applies, this scenario is modeled by the differential equation $y' = -2(y-20) + 6\delta(t-5)$ where t is in minutes and $t = 0$ corresponds to 12:00 PM (noon). Using Laplace transforms, find the temperature of Sid's pop tarts at 12:04 PM and 12:06 PM. Zero credit for using any other method of solution.

6. [2360/042726 (21 pts)] Matrices, $\mathbf{A}$, in linear systems of the form $\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}}$ are given. For each part, classify the geometry and stability of the fixed point(s) and choose the correct phase portrait. Write NONE if no phase portrait matches. No work need be shown.

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 1 \\ -4 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 1 \\ -4 & -2 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ (f) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ (g) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$



7. [2360/042726 (18 pts)] Let $\mathbf{A} = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) (3 pts) Show that the eigenvalues of $\mathbf{A}$ are $0, 1, 4$ by finding the roots of the characteristic equation.

(b) (15 pts) Use techniques shown in Homework12 to solve the variable coefficient initial value problem

$$t\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}},\ t > 0,\ \vec{\mathbf{x}}(1) = \begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}$$

writing your answer as a single vector. Use Cramer's Rule (zero credit for using a different method) to solve any linear system of algebraic equations that arise.

8. [2360/042726 (23 pts)] A horizontally oriented harmonic oscillator is governed by the equation $2\ddot{x} + 8\dot{x} + 26x = 0$. The motion is started by displacing the mass 3 units to the right of the equilibrium position and releasing it from rest.

(a) (2 pts) Give two reasons why the oscillator is not in resonance.

(b) (4 pts) Show that the initial value problem is equivalent to the linear system $\vec{\mathbf{u}}' = \begin{bmatrix} 0 & 1 \\ -13 & -4 \end{bmatrix}\vec{\mathbf{u}},\ \vec{\mathbf{u}}(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ where $\vec{\mathbf{u}}$ is an unknown vector.

(c) (15 pts) Solve the linear system from part (b). Zero credit for solving the original, given initial value problem. Hint: the final answer contains no fractions.

(d) (2 pts) Without computing any derivatives, what is the velocity of the mass when t is an integer multiple of π? Justify your answer.

Short table of Laplace Transforms: $\mathscr{L}\{f(t)\} = F(s) \equiv \int_0^\infty e^{-st} f(t)\,\mathrm{d}t$

In this table, a, b, c are real numbers with $c \geq 0$, and $n = 0, 1, 2, 3, \ldots$

$$\mathscr{L}\left\{t^n e^{at}\right\} = \frac{n!}{(s-a)^{n+1}} \qquad \mathscr{L}\left\{e^{at}\cos bt\right\} = \frac{s-a}{(s-a)^2+b^2} \qquad \mathscr{L}\left\{e^{at}\sin bt\right\} = \frac{b}{(s-a)^2+b^2}$$

$$\mathscr{L}\{\cosh bt\} = \frac{s}{s^2-b^2} \qquad \mathscr{L}\{\sinh bt\} = \frac{b}{s^2-b^2}$$

$$\mathscr{L}\{t^n f(t)\} = (-1)^n \frac{\mathrm{d}^n F(s)}{\mathrm{d}s^n} \qquad \mathscr{L}\left\{e^{at} f(t)\right\} = F(s-a) \qquad \mathscr{L}\{\delta(t-c)\} = e^{-cs}$$

$$\mathscr{L}\{t f'(t)\} = -F(s) - s\frac{\mathrm{d}F(s)}{\mathrm{d}s} \qquad \mathscr{L}\{f(t-c)\,\text{step}(t-c)\} = e^{-cs}F(s) \qquad \mathscr{L}\{f(t)\,\text{step}(t-c)\} = e^{-cs}\mathscr{L}\{f(t+c)\}$$

$$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{(n-1)}(0)$$
