1. (34 pts) Determine whether each of the following series converge or diverge. On this exam, state the name of any test you use, and verify that the conditions for using that test are satisfied.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(1+\ln(n)^2)}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 10^n}{n4^{2n+1}}$

2. (14 pts) Find the center, radius of convergence, and interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2\sqrt{n}+2}.$$

3. (16 pts) In the following two problems, use $f(x) = \dfrac{\sin(3x^2)}{x}$. You may assume that $f(0) = 0$, which makes $f(x)$ continuous and differentiable.

   (a) Find a Maclaurin series representation of $f(x)$, and determine its radius of convergence.

   (b) Find the value of $f^{(9)}(0)$, meaning the value of the ninth derivative of $f(x)$ at $x = 0$. You do not need to simplify your answer.

4. (22 pts) The following problems are not related.

   (a) Consider the function $h'(x) = \dfrac{x}{1+9x^2}$.

      i. Find a power series representation for $h'(x)$.

      ii. Use the power series from part (i) to find a power series representation for $g(x) = 4 \cdot h(x)$, assuming that $h(0) = 1$.

   (b) Determine whether $\displaystyle\sum_{n=0}^{\infty} \frac{-2}{(n+1)(n+2)}$ converges or diverges. If it converges, find its sum.

5. (14 pts) The following two problems are related:

   (a) Show that $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} e^{-2n+1}$ converges.

   (b) How many terms of the series in (a) are needed to guarantee a sum within 0.01 of the true value? You should solve for n, but you do not need to simplify any numerical expression.