1. For $P(x) = 2x^4 + 12x^3 + 18x^2$ answer the following. (12 pts)

   (a) Indicate on a graph or use arrow notation to indicate the end behavior of $P(x)$.

   Solution:

   $P(x) \to \boxed{\infty}$ as $x \to \infty$ and $P(x) \to \boxed{\infty}$ as $x \to -\infty$.

   (b) Find the y-intercept of $P(x)$.

   Solution:

   We set $x = 0$ and we find the y-intercept: $P(0) = 2 \cdot 0^4 + 12 \cdot 0^3 + 18 \cdot 0^2 = \boxed{0}$.

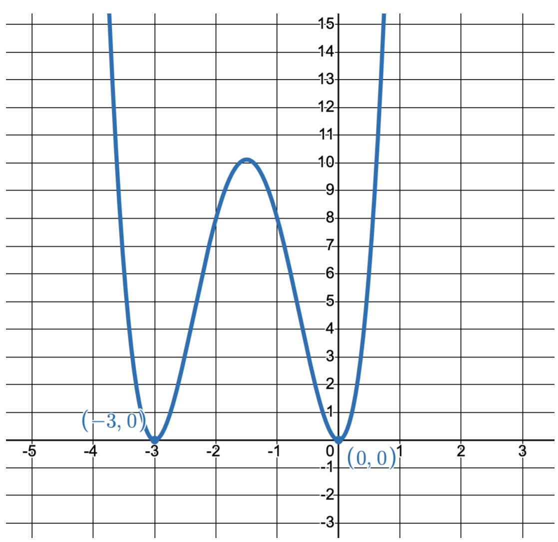
   (c) Find all zeros of $P(x)$ **and** identify the multiplicity of each zero.

   We set $P(x) = 0$ and we find the zeros.

$$2x^4 + 12x^3 + 18x^2 = 0 \quad (1)$$
$$2x^2\left(x^2 + 6x + 9\right) = 0 \quad (2)$$
$$2x^2(x + 3)(x + 3) = 0 \quad (3)$$
$$2x^2(x + 3)^2 = 0 \quad (4)$$

   So we get zeros $x = \boxed{0}$ and $x = \boxed{-3}$ each with multiplicity $\boxed{2}$. This means the graph will bounce (touch but not cross) the x-axis at each zero.

   (d) Sketch the shape of the graph of $P(x)$ using parts (a) - (c).



2. Find the quotient and remainder using long division: $\dfrac{-3x^4+6x^3+4x^2-2}{x^3-4x^2-x}$. (5 pts)

Solution:

We use long division to find the quotient and the remainder:

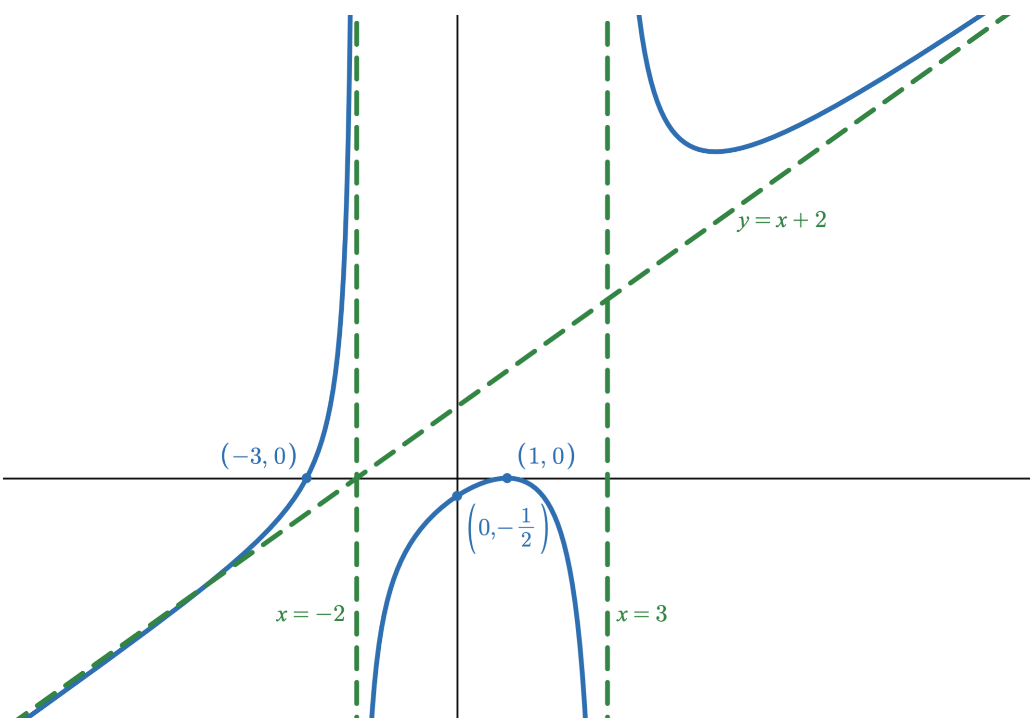
$$\begin{array}{r|rrrrr} & -3x & -6 & & & \\ \hline x^3-4x^2-x) & -3x^4 & +6x^3 & +4x^2 & +0x & -2 \\ & -(-3x^4 & +12x^3 & +3x^2) & & \\ \hline & & -6x^3 & +x^2 & +0x & -2 \\ & & -(-6x^3 & +24x^2 & +6x) & \\ \hline & & & -23x^2 & -6x & -2 \end{array}$$

Thus the quotient is $\boxed{Q(x)=-3x-6}$ and the remainder is $\boxed{R(x)=-23x^2-6x-2}$

3. Sketch the shape of the graph of a rational function, $g(x)$, that satisfies **all** of the information. **Label** all intercepts on the graph. (5 pts)

   i. The graph has y-intercept $\left(0,-\dfrac{1}{2}\right)$.
   ii. The graph has vertical asymptotes $x=-2$ and $x=3$.
   iii. The graph has a slant asymptote of $y=x+2$
   iv. The graph crosses at $(-3,0)$ and bounces (touches but does not cross) at $(1,0)$.
   v. The graph has no other x-intercepts.

Solution:

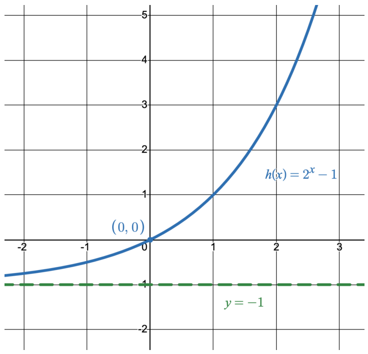


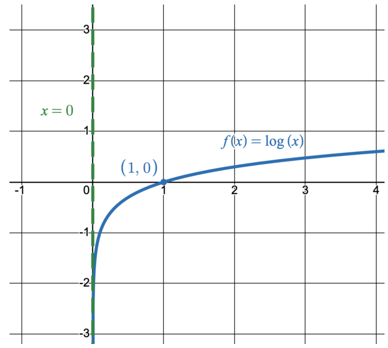
4. Sketch the following graphs: **Be sure to label** any asymptotes and intercepts for each graph.

   (a) $h(x) = 2^x - 1$ (3 pts)

   (b) $f(x) = \log(x)$ (3 pts)

   Solution:





   (c) Fill in the blanks for $h(x)$ from part (a): $h(x) \to$ ____ as $x \to -\infty$ and $h(x) \to$ ____ as $x \to \infty$. (2 pts)

   Solution:

   $h(x) \to \boxed{-1}$ as $x \to -\infty$ and $h(x) \to \boxed{\infty}$ as $x \to \infty$

5. Simplify: $5\left(3^x\right)^4 - 3^x\left(3^{3x} - 3^{-x}\right)$ (4 pts)

   Solution:

$$\begin{aligned}
5\left(3^x\right)^4 - 3^x\left(3^{3x} - 3^{-x}\right) &= 5 \cdot 3^{4x} - 3^x 3^{3x} + 3^x 3^{-x} && (5)\\
&= 5 \cdot 3^{4x} - 3^{4x} + 3^{x+(-x)} && (6)\\
&= 4 \cdot 3^{4x} + 3^0 && (7)\\
&= \boxed{4 \cdot 3^{4x} + 1} && (8)
\end{aligned}$$

6. (a) Simplify (rewrite without logs): $-\log_4(64) + \log\left(10^{\sqrt{2}}\right) + \log_6\left(\frac{1}{\sqrt{6}}\right) - e^{\ln(3)}$. (5 pts)

   Solution:

$$\begin{aligned}
-\log_4(64) + \log\left(10^{\sqrt{2}}\right) + \log_6\left(\frac{1}{\sqrt{6}}\right) - e^{\ln(3)} &= -3 + \sqrt{2} + \log_6\left(\frac{1}{6^{1/2}}\right) - 3 && (9)\\
&= -6 + \sqrt{2} + \log_6\left(6^{-1/2}\right) && (10)\\
&= -6 + \sqrt{2} - \frac{1}{2} && (11)\\
&= \boxed{-\frac{13}{2} + \sqrt{2}} && (12)
\end{aligned}$$

(b) Rewrite as a single logarithm without negative exponents: (5 pts)

$3\ln\left(y^2\right) - 4\ln(y) - \frac{2}{3}\ln(y)$

Solution:

$$\begin{aligned}
3\ln\left(y^2\right) - 4\ln(y) - \frac{2}{3}\ln(y) &= \ln\left(\left(y^2\right)^3\right) - \ln\left(y^4\right) - \ln\left(y^{\frac{2}{3}}\right) && (13)\\
&= \ln\left(y^6\right) - \ln\left(y^4\right) - \ln\left(y^{\frac{2}{3}}\right) && (14)\\
&= \ln\left(\frac{y^6}{y^4}\right) - \ln\left(y^{\frac{2}{3}}\right) && (15)\\
&= \ln\left(y^2\right) - \ln\left(y^{\frac{2}{3}}\right) && (16)\\
&= \ln\left(\frac{y^2}{y^{\frac{2}{3}}}\right) && (17)\\
&= \boxed{\ln\left(y^{\frac{4}{3}}\right)} && (18)
\end{aligned}$$

7. Solve the following equations for x. If there are no solutions write "no solutions" (be sure to justify answer for full credit). Recall you are not allowed a calculator, do not attempt to give a decimal answer. (16 pts)

(a) $3^{5x-1} = 9$

Solution:

$$\begin{aligned}
3^{5x-1} &= 9 && (19)\\
3^{5x-1} &= 3^2 && (20)\\
5x - 1 &= 2 && (21)\\
5x &= 3 && (22)\\
x &= \boxed{\frac{3}{5}} && (23)
\end{aligned}$$

(b) $7 = 2^{3+2x}$

Solution:

$$\begin{aligned}
7 &= 2^{3+2x} && (24)\\
\log_2(7) &= 3 + 2x && (25)\\
\log_2(7) - 3 &= 2x && (26)\\
x &= \boxed{\frac{\log_2(7) - 3}{2}} && (27)
\end{aligned}$$

(c) $\log(2x) = \log\left(3x^2 - 9\right) - \log(3)$

Solution:

$$\log(2x) = \log(3x^2 - 9) - \log(3) \tag{28}$$
$$\log(2x) = \log\left(\frac{3x^2 - 9}{3}\right) \tag{29}$$
$$\log(2x) = \log(x^2 - 3) \tag{30}$$
$$2x = x^2 - 3 \tag{31}$$
$$0 = x^2 - 2x - 3 \tag{32}$$
$$0 = (x - 3)(x + 1) \tag{33}$$
$$x = 3, -1 \tag{34}$$

Checking in the original equation:

$$x = 3: \quad \log(6) = \log(18) - \log(3) = \log(6)$$

so $x = 3$ works.

$$x = -1: \quad \log(2(-1)) = \log(-2)$$

which is undefined, so $x = -1$ does not work.

$$\boxed{x = 3}$$

(d) $5 = 2\log_2\left(x - \frac{1}{2}\right)$

Solution:

$$5 = 2\log_2\left(x - \frac{1}{2}\right) \tag{35}$$
$$\frac{5}{2} = \log_2\left(x - \frac{1}{2}\right) \tag{36}$$
$$2^{5/2} = x - \frac{1}{2} \tag{37}$$
$$x = \frac{1}{2} + 2^{5/2} \tag{38}$$
$$x = \boxed{\frac{1}{2} + 4\sqrt{2}} \tag{39}$$

8. The raccoon population in a certain Colorado town was 100 in 2022. The population is expected to double every 5 years. Use this information to answer the following.

(a) Find an exponential model $n(t) = n_\circ 2^{t/a}$ for the number of raccoons after t years. (3 pts)

Solution:

$$n(t) = n_0 2^{t/a} \tag{40}$$
$$n_0 = 100 \tag{41}$$
$$a = 5 \tag{42}$$
$$n(t) = \boxed{100 \cdot 2^{t/5}} \tag{43}$$

(b) What is the expected population size in 2032? Give the exact answer. Recall you are not allowed a calculator, do not attempt to give a decimal answer. (3 pts)

Solution:

$$n(t) = 100 \cdot 2^{t/5} \tag{44}$$
$$n(10) = 100 \cdot 2^{10/5} \tag{45}$$
$$= 100 \cdot 2^2 \tag{46}$$
$$n(10) = \boxed{400} \tag{47}$$

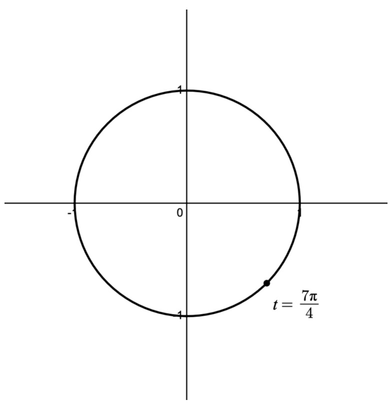
(c) How many years until the population triples? Give the exact answer. Recall you are not allowed a calculator, do not attempt to give a decimal answer. (4 pts)

Solution:

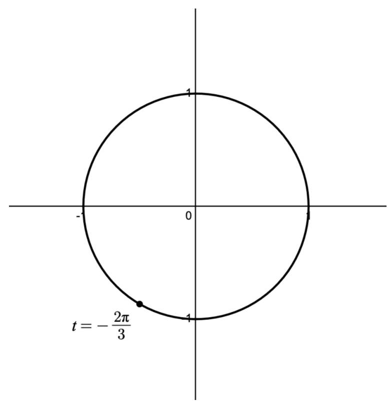
$$100 \cdot 2^{t/5} = 300 \quad (48)$$
$$2^{t/5} = 3 \quad (49)$$
$$\log_2\left(2^{t/5}\right) = \log_2(3) \quad (50)$$
$$\frac{t}{5} = \log_2(3) \quad (51)$$
$$t = \boxed{5\log_2(3)} \quad (52)$$

9. Graph the terminal point determined by each real number on the unit circle.

(a) $t = \frac{7\pi}{4}$ (2 pts)



(b) $t = -\frac{2\pi}{3}$ (2 pts)



10. For the real number t, suppose we know $\sin(t) > 0$ and $\cos(t) < 0$. What quadrant does the terminal point determined by t lie in? (3 pts)

Solution:

Since $\sin(t)$ is positive, the terminal point is above the x-axis.

Since $\cos(t)$ is negative, the terminal point is to the left of the y-axis.

Therefore, the terminal point lies in $\boxed{\text{Quadrant II}}$.

11. If we know $\sin(t) = -\frac{2}{5}$ what is $\csc(t)$? (3 pts)

Solution:

$$\sin(t) = -\frac{2}{5} \quad (53)$$
$$\csc(t) = \frac{1}{\sin(t)} \quad (54)$$
$$= \frac{1}{-\frac{2}{5}} \quad (55)$$
$$\csc(t) = \boxed{-\frac{5}{2}} \quad (56)$$

12. If we know the terminal point on the unit circle, $\left(\frac{\sqrt{3}}{8}, y\right)$, is determined by t and we know $\tan(t) > 0$, find y. (5 pts)

Solution:

$$x^2 + y^2 = 1 \tag{57}$$

$$\left(\frac{\sqrt{3}}{8}\right)^2 + y^2 = 1 \tag{58}$$

$$\frac{3}{64} + y^2 = 1 \tag{59}$$

$$y^2 = \frac{61}{64} \tag{60}$$

$$y = \pm\frac{\sqrt{61}}{8} \tag{61}$$

Since $\tan(t) = \frac{y}{x} > 0$ and $x = \frac{\sqrt{3}}{8} > 0$, we must have $y > 0$.

Therefore,

$$\boxed{y = \frac{\sqrt{61}}{8}}$$

13. Find the exact value of each of the following. If a value does not exist write DNE. (15 pts)

(a) $\sin(\pi)$

Solution:

$$\sin(\pi) = 0$$

(b) $\cos\left(\frac{4\pi}{3}\right)$

Solution:

$$\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

(c) $\tan\left(-\frac{5\pi}{6}\right)$

Solution:

$$\tan\left(-\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

(d) $\sec(0)$

Solution:

$$\sec(0) = 1$$

(e) $\cos\left(\frac{7\pi}{4}\right)$

Solution:

$$\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$$