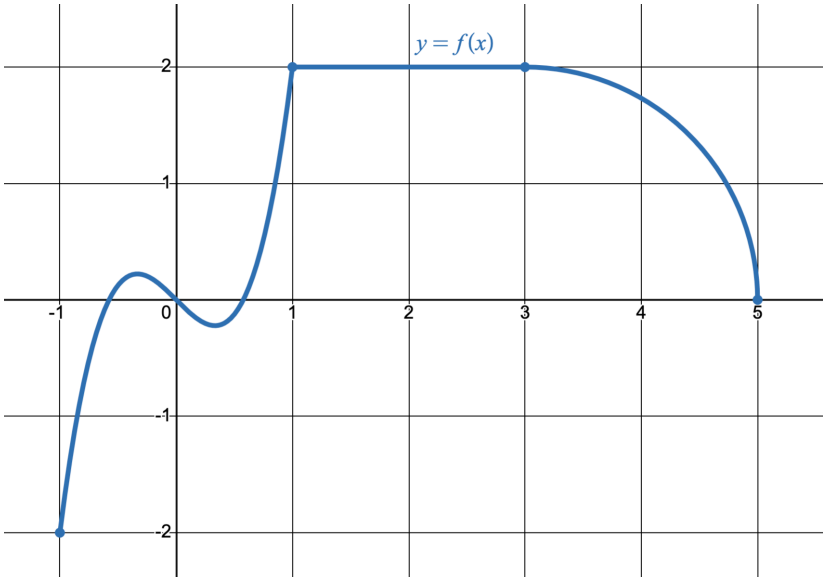
1. (32 points) The following parts are not related.

   (a) Evaluate: $\displaystyle\int \left[\frac{\sqrt{x}}{x^3} + 2\sec(x)\tan(x)\right]\, dx$

   (b) Evaluate: $\displaystyle\frac{d}{dx}\int_3^{x^3} \frac{\tan\left(t^2\right)}{t^4}\, dt$

   (c) Evaluate: $\displaystyle\int_{-2}^{4} h(x)\, dx$ if we know $\displaystyle\int_{-2}^{4} (g(x)+h(x))\, dx = 4$ and $\displaystyle\int_4^{-2} 3g(x)\, dx = 8$

   (d) Evaluate: $\displaystyle\int_{-1}^{5} f(x)\, dx$ for the function $y = f(x)$ graphed below. Note that the graph consists of an odd function between $x = -1$ and $x = 1$, a line segment between $x = 1$ and $x = 3$, and a quarter circle between $x = 3$ and $x = 5$. You may use geometry or a combination of geometry and integration to compute your answer.



2. (20 points) Consider a function $f(x) = x^2 - 4x - 5$, which is referenced in each part of this problem.

   (a) Verify that $f(x)$ satisfies the hypotheses of Rolle's Theorem on $[-1, 5]$. Then, determine the value(s) of c at which $f'(c) = 0$ in $(-1, 5)$.

   (b) Assuming that $f(x)$ satisfies the hypotheses of the Mean Value Theorem on $[1, 4]$, determine all value(s) of c in $(1, 4)$ that satisfy the conclusion of the Mean Value Theorem.

3. (22 points) Consider the function

$$f(x) = \frac{1}{x(x-3)^2} \text{ with } f'(x) = \frac{3(1-x)}{x^2(x-3)^3} \text{ and } f''(x) = \frac{6(2x^2-4x+3)}{x^3(x-3)^4}$$

   (a) Does the function have vertical asymptotes? Justify your answer using appropriate limits.

   (b) Does the function have horizontal asymptotes? Justify your answer using appropriate limits.

   (c) Find the x coordinates of the local maxima and minima, if any.

   (d) Find the interval(s) where $f(x)$ is concave up, and the interval(s) where it is concave down. Hint: The numerator of $f''(x)$ cannot be factored in the real numbers and is always positive.

   (e) Sketch a graph of $f(x)$. Clearly label intercepts, asymptotes, and local extrema.

4. (14 points) All questions below will consider the function $f(x) = \sin x + x - 1$.

   (a) Given an initial guess of $x_1 = \frac{\pi}{2}$, compute the first two iterations of Newton's method to find the root of $f(x)$. In other words, find x_3.

   (b) What is the Riemann sum with n rectangles that would be used to approximate the area between $f(x)$ and the x-axis on the interval from $[0, \pi]$. Leave your answer in terms of a summation (sigma notation). Make sure to explicitly define Δx and $f(x_i)$ for this particular function.

5. (12 points) Suppose an airline policy states that all baggage must be box-shaped with a sum of the length, width, and height not exceeding 64 inches. What are the **dimensions** and **volume** of a square-based box (i.e. the base of the box is square) with the greatest volume under these conditions? Make sure to label the dimensions of the box and to report the maximum volume. You must use calculus and the optimization methods covered in our class to solve this problem. Show all work and box your answers.

END OF TEST