

Write your name and your professor's name or your section number below. You are *not* allowed to use textbooks, notes, or a calculator. To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise.

This exam has 5 problems and is worth 100 points.

Name:

Instructor and Section:

1. (24 pts, 6 each) If the statement is **always true**, write “TRUE”; if it is possible for the statement to be false then mark “FALSE.” You must give a **justification** for your answer. That is, if the answer is true, provide a brief proof. If the answer is false, provide a counterexample.
- (a) If A is square and $\det A = 0$ then the equation $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.
 - (b) Multiplying a column of a matrix by a real number also multiplies the determinant by the same real number.
 - (c) The set of 2×2 matrices with a 0 in the 2, 1 entry is a subspace of $\mathbb{R}^{2 \times 2}$ with dimension 3.
 - (d) Let A be a 4×4 matrix. If $A^2 = A$, then either $A = I$ or $A = 0$ (the zero matrix).

2. (20 pts) Let

$$A = \begin{pmatrix} -1 & 2 & 8 \\ 2 & 3 & 5 \\ 0 & -1 & -3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -5 \\ 3 \\ -1 \end{pmatrix}$$

- (a) (8 pts) Find the Row Echelon Form of A
- (b) (2 pts) What is the rank of A ?
- (c) (6 pts) Does $A\mathbf{x} = \mathbf{b}$ have a solution? If so, is there more than one solution?
- (d) (4 pts) Are there any vectors $\mathbf{c}$ such that $A\mathbf{x} = \mathbf{c}$ has infinitely many solutions?

3. (15 pts) Consider the set of vectors

$$V = \left\{ \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 11 \end{pmatrix} \right\}.$$

- (a) (7 points) Are these vectors linearly dependent or linearly independent? Justify your answer.
- (b) (4 points) Find a basis for the span of V and state its dimension.
- (c) (4 points) Is the vector $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$ in the span of V ?

4. (20 pts) The following questions are unrelated.

- (a) (5 pts) For a regular matrix A that admits an LU decomposition, what is the determinant of UL^T ?
- (b) (10 pts) Consider the matrix

$$B = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Find the LDL^T decomposition of B .

- (c) (5 pts) For the following system

$$P \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 5 \\ 2 \\ 4 \end{pmatrix},$$

what is a matrix P that makes this equation true?

5. (21 pts)

Consider matrix

$$A = \begin{pmatrix} 0 & r & -s \\ -r & 0 & t \\ s & -t & 0 \end{pmatrix}$$

and a linear system

$$A\mathbf{x} = \mathbf{b},$$

where

$$\mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

- (a) (7 pts) Is the matrix A invertible? Justify your answer.
- (b) (7 pts) What is the rank of A if $r = s = t = 1$? Justify your answer.
- (c) (7 pts) Find the general solution of $A\mathbf{x} = \mathbf{b}$ assuming $r = s = t = 1$.