1. An unusual region has its elevation given by $z = f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2$, where x, y, and z are all measured in meters.

   (a) (14 points) Determine the (x, y)-coordinates of any peaks (local maximums), valleys (local minimums), or saddle points. (Be sure to determine which of the three occurs at each such point.)

   (b) (9 points) A small South American deer called a *pudu* stands at the point $(2, 1)$. From this point, what is the **unit vector** that points in the direction in which the pudu should head so its path will be the steepest uphill climb?

   (c) (9 points) The pudu starts to travel such that the (x, y)-coordinates of its location are given by

   $$\mathbf{r}(t) = \langle 2 - t, 1 - \frac{1}{2}t^2 \rangle$$

   after t minutes.

   i. Find the rate of change of the pudu's elevation with respect to *time* when it is at the point $(0, -1)$.

   ii. Find the rate of change of the pudu's elevation with respect to *distance* when it is at the point $(0, -1)$.

   iii. Is the pudu heading **uphill** or **downhill** at the point $(0, -1)$?

2. Recall that the volume of a right circular cone is given by $V(r, h) = \frac{\pi}{3}r^2h$ where r is the radius and h is the height.

   (a) (9 points) Determine the linear approximation of $V(r, h)$ centered at $(r, h) = (3, 4)$.

   (b) (4 points) Use your answer from (a) to approximate the volume of a right circular cone whose radius is 3.1 cm and height is 3.9 cm.

   (c) (9 points) Use Taylor's Theorem to determine an upper bound on the error in approximating the volume of the cone with the linear approximation when $|r - 3| < 0.1$ and $|h - 4| < 0.1$. (For this problem only, you do not need to fully simplify your final answer, but it should be in a form that could be directly input into a simple calculator. To earn points on this problem you must use the techniques from class with Taylor's Theorem.)

   (d) (9 points) Assume a cone has a radius of $r = 3$ cm and a height of $h = 4$ cm. It is exposed to some radioactive material and its dimensions start to change. If its volume remains unchanged but the height starts growing at a rate of 2 cm per hour, determine the rate of change of its radius. *Include the correct units in your final answer.*

3. The following two problems are not related.

   (a) (9 points) Evaluate the following limit or show it does not exist: $\displaystyle\lim_{(x,y)\to(0,1)} \frac{xy}{x + (y - 1)^2}$

   (b) (9 points) Let $f(x, y)$ be defined by

   $$f(x, y) = \begin{cases} \dfrac{(2x + 4)^2 - (y - 5)^2}{2x - y + 9}, & (x, y) \neq (-4, 1) \\ c, & (x, y) = (-4, 1) \end{cases}$$

   Is there a value of c that makes $f(x, y)$ continuous at $(-4, 1)$? If so, find the value of c. If not, explain why not. (In either case be sure to justify your answer with the definition of continuity)

4. Consider the function $f(x, y) = e^{x^2 + 2x + y^2 - 4y}$.

(a) (9 points) Use Lagrange Multipliers to determine both the **absolute maximum value** and **absolute minimum value** of $f(x, y)$ when $x^2 + y^2 = 20$.

(b) (3 points) Explain why the Extreme Value Theorem guarantees the existence of both absolute maximum and minimum values for $f(x, y)$ when $x^2 + y^2 \leq 20$.

(c) (7 points) Determine the **absolute maximum value** and **absolute minimum value** of $f(x, y)$ when $x^2 + y^2 \leq 20$.