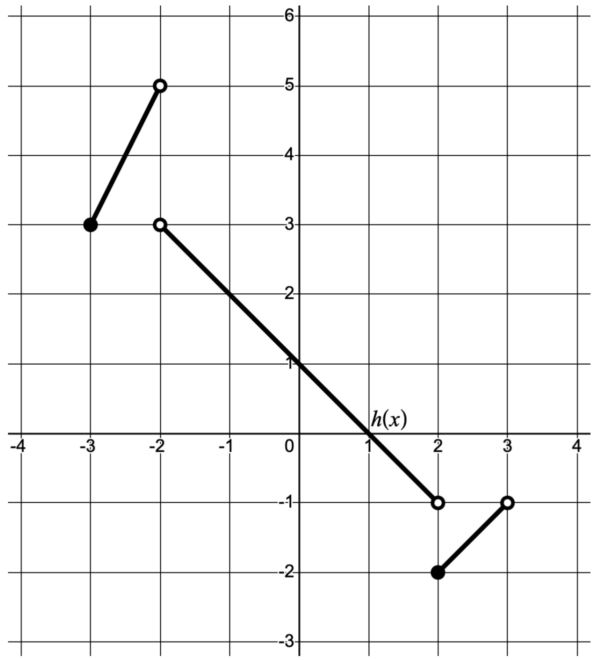
1. Use the table of values of $p(x)$ and the graph of $h(x)$ (which consists of all three line segments) provided here to answer each of the problems below. No justification is required for this problem (though justification can help earn potential partial credit). (16 pts):

x	$p(x)$
-1	-2
0	2
1	-2
2	-3
3	5



(a) Identify the domain of $h(x)$.
Solution: $\boxed{[-3,-2) \cup (-2,3)}$

(b) Identify the the range of $h(x)$.
Solution: $\boxed{[-2,-1) \cup (-1,5)}$

(c) Find $(h+p)(0)$.
Solution: $(h+p)(0) = h(0) + p(0) = 1 + 2 = \boxed{3}$

(d) If we know $p(x)$ is an even function find the value $p(-3)$.
Solution: If p is even, then $p(-3) = p(3) = \boxed{5}$

(e) Find $(h \circ h)(-1)$.
Solution: $(h \circ h)(-1) = h(h(-1)) = h(2) = \boxed{-2}$

(f) Is $h(x)$ one-to-one? Give a brief sentence why $h(x)$ is or isn't one-to-one.
Solution: Yes, $h(x)$ is one-to-one as each y-value comes from only one x-value, the graph of $h(x)$ passes the horizontal line test.

(g) Find the x-values where $h(x) \leq 0$. Give your answer in interval notation.
Solution: $\boxed{[1,3)}$

(h) Solve the equation $h(x) = 3$.
Solution: $\boxed{x = -3}$

2. Solve the following inequalities. **Justify your answers by using a number line or sign chart when appropriate**. Answers without full justification will not receive full credit. Express all answers in interval notation. (11 pts)

(a) $\frac{1}{2}x - \frac{5}{12} < \frac{1}{3}$

Solution:

$$\frac{1}{2}x < \frac{1}{3} + \frac{5}{12} \tag{1}$$
$$12 \cdot \left(\frac{1}{2}x\right) < 12 \cdot \left(\frac{1}{3} + \frac{5}{12}\right) \tag{2}$$
$$6x < 4 + 5 \tag{3}$$
$$6x < 9 \tag{4}$$
$$x < \frac{3}{2} \tag{5}$$

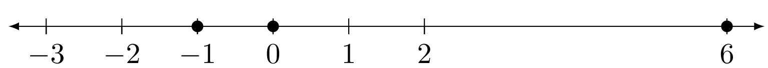
$$\boxed{(-\infty, \tfrac{3}{2})}$$

(b) $x^4 - 5x^3 - 6x^2 > 0$

Solution:

$$x^4 - 5x^3 - 6x^2 > 0 \tag{6}$$
$$x^2(x^2 - 5x - 6) > 0 \tag{7}$$
$$x^2(x-6)(x+1) > 0 \tag{8}$$

The values that make the left side 0 are $x = -1, 0, 6$. We place them on the number line:



Using test values: $x = -2, -0.5, 7$ we see $x^2(x-6)(x+1)$ is positive on the intervals $(-\infty, -1)$ and $(6, \infty)$ so the answer is:

$$\boxed{(-\infty, -1) \cup (6, \infty)}$$

(c) $\frac{-4}{x+2} \leq 0$

For this problem when the denominator is positive then $\frac{-4}{x+2}$ is negative:

$$x + 2 > 0$$

$$x > -2$$

$$\boxed{(-2, \infty)}$$

3. Solve the following equation: (4 pts)

$$|2x-1|=4$$

Solution:

$$2x-1=\pm 4$$

$$2x-1=4 \quad\Rightarrow\quad 2x=5 \quad\Rightarrow\quad x=\frac{5}{2}$$

$$2x-1=-4 \quad\Rightarrow\quad 2x=-3 \quad\Rightarrow\quad x=-\frac{3}{2}$$

$$\boxed{x=\frac{5}{2} \text{ or } x=-\frac{3}{2}}$$

4. Solve the following equation: (5 pts)

$$\frac{1}{3(x+1)}+\frac{x+2}{x^2+x}=-\frac{5}{3x}$$

Solution:

$$\frac{1}{3(x+1)}+\frac{x+2}{x^2+x}=-\frac{5}{3x} \tag{9}$$

$$\frac{1}{3(x+1)}+\frac{x+2}{x(x+1)}=-\frac{5}{3x} \tag{10}$$

$$3x(x+1)\cdot\left(\frac{1}{3(x+1)}+\frac{x+2}{x(x+1)}\right)=3x(x+1)\cdot\left(-\frac{5}{3x}\right) \tag{11}$$

$$3x(x+1)\cdot\frac{1}{3(x+1)}+3x(x+1)\cdot\frac{x+2}{x(x+1)}=3x(x+1)\cdot\left(-\frac{5}{3x}\right) \tag{12}$$

$$x+3(x+2)=-(x+1)5 \tag{13}$$

$$x+3x+6=-5x-5 \tag{14}$$

$$9x=-11 \tag{15}$$

$$x=\boxed{-\frac{11}{9}} \tag{16}$$

5. The following are unrelated: (6 pts)

   (a) Find the equation of the line parallel to the x-axis and through the point $(-1, 4)$.
   Solution:
   A line parallel to the x-axis is horizontal, and since the line passes through $(-1, 4)$, the y-value is always 4. Therefore,

   $$\boxed{y = 4}$$

   (b) Find all value(s) of c such that the distance between the two points, $(4, c)$ and $(2, 0)$, is 3.

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

   $$\sqrt{(4-2)^2 + (c-0)^2} = 3$$

   $$\sqrt{2^2 + c^2} = 3$$

   $$\sqrt{4 + c^2} = 3$$

   $$4 + c^2 = 9$$

   $$c^2 = 5$$

   $$\boxed{c = \pm\sqrt{5}}$$

6. Biologists have observed that the movement speed, v measured in inches per second, of a certain species of lizard depends on temperature, t measured in degrees Fahrenheit. The relationship appears to be very nearly linear. A lizard moves 4 inches per second at 50° F and 6 inches per second at 70°F. (7 pts)

   (a) Find the slope of the linear equation that relates the temperature t and the speed v.
   Solution:

   $$m = \frac{\Delta v}{\Delta t} = \frac{6-4}{70-50} = \frac{2}{20} = \frac{1}{10}$$

   $$\boxed{m = \frac{1}{10}}$$

   (b) What are the units of the slope of the linear equation that relates the temperature t and the speed v.

   Solution:

   The slope is given by: $\dfrac{\text{change in speed}}{\text{change in temperature}}$ so the units of slope are

   $\boxed{\dfrac{\text{inches per second}}{\text{degree Fahrenheit}}}$

   (or $\boxed{\text{(in/s)}/^\circ F}$).

(c) Find the linear equation that relates the temperature t and the speed v.

Solution:
Point-slope form with point $(50, 4)$:

$$\boxed{v - 4 = \frac{1}{10}(t - 50)}$$

or

$$v - 4 = \frac{1}{10}t - 5$$
$$v = \frac{1}{10}t - 1$$

In slope-intercept form we get:

$$\boxed{v = \frac{1}{10}t - 1}$$

7. Find the center and radius of the circle given by: $x^2 - 4x + y^2 = 5$. (4 pts)

Solution:

$$x^2 - 4x + y^2 = 5 \tag{17}$$
$$x^2 - 4x + \_\_ + y^2 = 5 + \_\_ \tag{18}$$
$$x^2 - 4x + 2^2 + y^2 = 5 + 2^2 \tag{19}$$
$$(x-2)^2 + y^2 = 9 \tag{20}$$

So the center of the circle is located at $\boxed{(2, 0)}$ and the radius is $\boxed{3}$.

8. Find the domain of the following functions. Express your answers in interval notation. (12 pts)

(a) $n(x) = x - |5x| + 13$

Solution:

There are no even roots and no potential for division by zero. The domain is $\boxed{(-\infty, \infty)}$.

(b) $s(x) = \dfrac{x^2 - 4x + 3}{2x^2 - 6x}$

Solution:

We must exclude all x-values that make the denominator zero. We do this by first finding the x-values that result in division by zero.

$$2x^2 - 6x = 0 \tag{21}$$
$$x(x-3) = 0 \tag{22}$$

Therefore the domain requires that $x \neq 0, 3$. The domain is $\boxed{(-\infty, 0) \cup (0, 3) \cup (3, \infty)}$.

(c) $h(x) = \dfrac{x\sqrt{5-3x}}{4-x^2}$

Solution:

First, we must find and exclude all x-values that make the denominator equal to zero.

$$\begin{aligned} 4 - x^2 &= 0 && (23)\\ 4 &= x^2 && (24)\\ \pm 2 &= x && (25) \end{aligned}$$

So we need to exclude $x = 2$ and $x = -2$ from the domain.

Next, we cannot take the square root of a negative number. Therefore we need to find the x-values that result in $5 - 3x \geq 0$.

$$\begin{aligned} 5 - 3x &\geq 0 && (26)\\ -3x &\geq -5 && (27)\\ x &\leq \frac{5}{3} && (28) \end{aligned}$$

This restriction alone gives $\left(-\infty, \dfrac{5}{3}\right]$. So the domain must exclude -2 and also satisfy $\left(-\infty, \dfrac{5}{3}\right]$.

Therefore the domain is $\boxed{(-\infty,-2) \cup \left(-2, \dfrac{5}{3}\right)}$. Note that $\dfrac{5}{3} < 2$ since $2 = \dfrac{6}{3}$ and we can see that $\dfrac{5}{3} < \dfrac{6}{3}$. Thus, $x = 2$ is already excluded from the domain in the interval $\left(-\infty, \dfrac{5}{3}\right]$.

9. For $f(x) = \dfrac{x+5}{x-2}$ and $g(x) = x + \dfrac{5}{x} + 3$, find the following: (12 pts)

(a) Is $g(x)$ odd, even, or neither? As usual, fully justify your answer for credit.

Solution:

To check if $g(x)$ is odd, even, or neither we start by replacing x by $-x$ in $g(x)$.

$$\begin{aligned} g(-x) &= (-x) + \frac{5}{(-x)} + 3 && (29)\\ &= -x - \frac{5}{x} + 3 && (30) \end{aligned}$$

Since $g(-x) \neq g(x)$ and $g(-x) \neq -g(x)$ then $g(x)$ is $\boxed{\text{neither}}$ odd nor even.

(b) Find $h(x) = \left(\frac{f}{g}\right)(x)$ As usual, simplify your final answer.

Solution:

$$\begin{align}
h(x) &= \left(\frac{f}{g}\right)(x) \tag{31}\\
&= \frac{f(x)}{g(x)} \tag{32}\\
&= \frac{\frac{x+5}{x-2}}{x+\frac{5}{x}+3} \tag{33}\\
&= \frac{\frac{x+5}{x-2}}{\frac{x^2}{x}+\frac{5}{x}+\frac{3x}{x}} \tag{34}\\
&= \frac{\frac{x+5}{x-2}}{\frac{x^2+3x+5}{x}} \tag{35}\\
&= \frac{x+5}{x-2}\cdot\frac{x}{x^2+3x+5} \tag{36}\\
&= \boxed{\frac{x(x+5)}{(x-2)(x^2+3x+5)}} \tag{37}
\end{align}$$

(c) Find the domain of $h(x) = \left(\frac{f}{g}\right)(x)$. Express your answer in interval notation.

Solution:

While working out the solution for part (b) we see that when we write $\frac{\frac{x+5}{x-2}}{x+\frac{5}{x}+3}$ we need to exclude $x = 0$ and $x = 2$ in our final domain since both x-values result in division by zero. In the final answer we get $\frac{x(x+5)}{(x-2)(x^2+3x+5)}$. Setting the denominator equal to zero we get:

$$(x-2)(x^2+3x+5) = 0$$

By the multiplicative property of zero we see that $x - 2 = 0$ when $x = 2$ and attempting to solve $x^2 + 3x + 5 = 0$ by the quadratic formula results in:

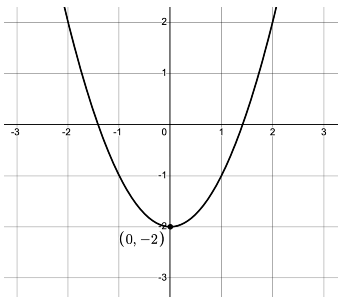
$$\begin{align}
x &= \frac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)} \tag{38}\\
x &= \frac{-3 \pm \sqrt{-11}}{2} \tag{39}
\end{align}$$

The solutions to $x^2 + 3x + 5 = 0$ are not real so do not affect domain. The resulting in domain $\boxed{(-\infty, 0) \cup (0, 2) \cup (2, \infty)}$.

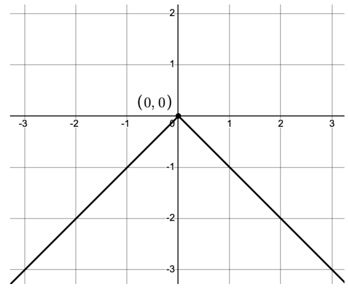
10. Sketch the shape of the graph of each of the following on the provided axes. **Label** all x and y-intercepts. (16 pts)

(a) $f(x) = x^2 - 2$

Solution:

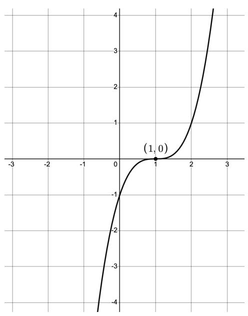


(b) $g(x) = -|x|$

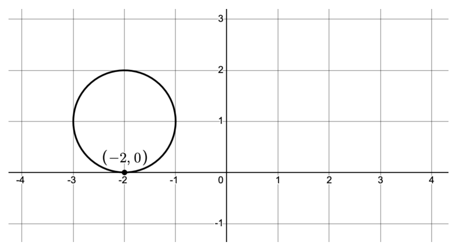


(c) $k(x) = (x - 1)^3$

Solution:

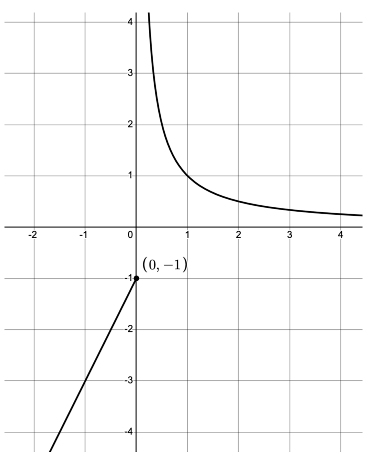


(d) $(x + 2)^2 + (y - 1)^2 = 1$

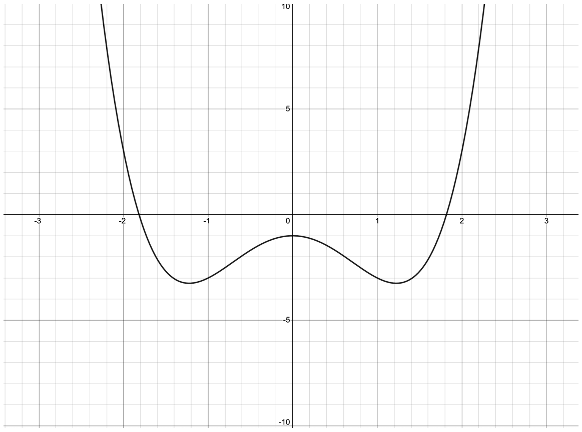


(e) $q(x) = \begin{cases} 2x - 1 & \text{if} \quad x \leq 0 \\ \frac{1}{x} & \text{if} \quad x > 0 \end{cases}$

Solution:



11. Given the graph of a function below, is the function odd, even, or neither? **No justification is needed.** (2 pts)



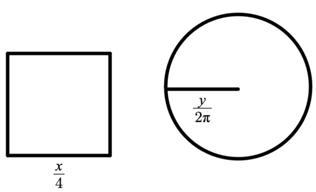
Solution:

The function whose graph is given is $\boxed{\text{even}}$ since the graph is symmetric about the y-axis.

12. A wire 13 inches long is cut into two pieces, the first piece has length x and the second piece has length y. The **first piece** is bent into the shape of a **square** and the **second piece** is bent into the shape of a **circle**. Express the total area of the square and circle as a function of x. (5 pts)

Solution:

First we draw a picture:



If the length of each side of a square is n then the perimeter formula gives $4n = x$ and $n = \dfrac{x}{4}$. In the diagram, we label each side of the square as $\dfrac{x}{4}$.

The circle has perimeter y so the radius of the circle satisfies $2\pi r = y$ and $r = \dfrac{y}{2\pi}$. In the diagram, we label the radius of the circle as $\dfrac{y}{2\pi}$.

The area of a square is $A = n^2$ and the area of a circle is $A = \pi r^2$ and thus the combined area of the square and circle is, $A = n^2 + \pi r^2 = \left(\frac{x}{4}\right)\left(\frac{x}{4}\right) + \pi\left(\frac{y}{2\pi}\right)^2$. We also know that $13 = x + y$ and $y = 13 - x$. So:

$$A = \left(\frac{x}{4}\right)^2 + \frac{y^2}{4\pi} \tag{40}$$

$$= \frac{x^2}{16} + \frac{(13-x)^2}{4\pi} \tag{41}$$

and the answer is $\boxed{A(x) = \dfrac{x^2}{16} + \dfrac{(13-x)^2}{4\pi}}$.