1. (16 pts) The following problems are unrelated.

   (a) Give the full partial fraction decomposition for the rational function

   $$\frac{1}{(x-2)(x^4-16)(x^2+x+5)^2}$$

   You do NOT need to solve for the unknown constants.

   (b) Assume that $a(x)$ and $b(x)$ are two continuous functions such that $0 \le a(x) \le b(x)$. If you say yes to the following questions, explain why and state any theorems you use. If you say no, give appropriate examples of functions $a(x)$ and $b(x)$.

   i. Suppose that $\displaystyle\int_1^\infty b(x)\,dx$ converges. Must $\displaystyle\int_1^\infty a(x)\,dx$ converge too?

   ii. Suppose instead that $\displaystyle\int_1^\infty b(x)\,dx$ diverges. Must $\displaystyle\int_1^\infty a(x)\,dx$ diverge too?

2. (36 pts) Evaluate the following expressions.

   (a) $\displaystyle\int \frac{3x-2}{x^2-2x+1}\,dx$

   (b) $\displaystyle\int \frac{x^2}{(1-x^2)^{5/2}}\,dx$ (*hint*: recall that $\displaystyle\int \sec^2(x)\,dx = \tan(x)$).

   (c) $\displaystyle\int e^x \sin(kx)\,dx$, where k is a constant.

3. (24 pts) Evaluate each expression below. If an integral converges, determine its value. If an integral diverges, justify why.

   (a) $\displaystyle\int_0^\infty \frac{dt}{(1+t^2)(1+\arctan(t))^2}$

   (b) $\displaystyle\int_0^3 \frac{x-1}{x^{4/3}}\,dx$

4. (12 pts) The table below records the velocity $v(t)$ in meters per second of a car at time t (measured in seconds):

t	0	2	4	6	8	10	12
$v(t)$	0	5.1	10.9	16.8	22.8	28.9	35.1

   (a) Give an approximation for the distance the car travels over the first 12 seconds using the trapezoidal rule with $n = 3$ (You do NOT need to simplify your expression, but you should give an expression which could be easily entered into a calculator).

   (b) Sort the values L_3, R_3, and T_3 in order from least to greatest. *Remember that these are the left-sided, right-sided, and trapezoidal approximations with* $n = 3$.

5. (12 pts) The function $f(x)$ has a second derivative given by

   $$f''(x) = \frac{x\sin(x) - \cos^2(x)}{1+x^3}.$$

   Determine how large n should be so that the midpoint approximation M_n for the integral $\displaystyle\int_2^3 f(x)\,dx$ is within 10^{-3} of the true value. *You should solve for* n*, but you do not need to simplify your answer.*