1. (18 points) Given the function:

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

Compute the following. If the limit does not exist, write DNE. Justification is not required for this problem.

(a) $\lim_{x \to 1^-} f(x)$ (b) $\lim_{x \to 1^+} f(x)$ (c) $f(1)$

(d) $\lim_{x \to 2^-} f(x)$ (e) $\lim_{x \to 2^+} f(x)$ (f) $\lim_{x \to 2} f(x)$

2. (24 pts) Evaluate the following limits and simplify your answers. To get full credit, be sure to show all justifications and intermediate steps. If a limit does not exist, clearly state this. If you use a theorem, clearly state its name and show that its hypothesis are satisfied. (You may not use L'Hospital's rule or dominance of powers arguments.):

(a) $\lim_{x \to 2} \dfrac{x - 2}{\sqrt{4x + 1} - 3}$

(b) $\lim_{t \to 0} \left(\dfrac{\sin(3t)}{5t^3 - 4t} \right)$

(c) $\lim_{x \to 0} x^4 \cos \dfrac{2}{x}$

3. (23 points) Let $f(x) = \cos(x) - \frac{1}{2}$

   (a) Give the domain and the range of $f(x)$ in interval notation.

   (b) Graph $f(x)$ on $[0, 2\pi]$. Be sure that you clearly label the x and y intercepts. Also, indicate where the function attains its minimum value on the interval.

   (c) Give the domain and the range of $|f(x)|$ in interval notation.

   (d) This part is unrelated to the above. Given the function $g(x) = 2\cos(x) + \sin(2x)$. Find where $g(x) \geq 0$ on the interval $[0, 2\pi]$.

4. (20 pts) Consider the function $f(x) = \dfrac{x^2 + x}{1 + x}$ and $g(x) = \sin(2x)$.

   (a) Is $f(x)$ an even function, an odd function, or neither? Justify your answer algebraically.

   (b) Give the domain of $f(x)$ and the domain of $g(x)$ in interval notation.

   (c) Find the values where $f(x)$ is discontinuous and state which type of discontinuity it is. **Make sure to justify your answer with the appropriate limit definitions.**

   (d) Find $(f \circ g)(x)$ and state its domain within the interval $[0, 2\pi]$. Give your answer in interval notation.

5. (15 pts) Consider the function:

$$f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ a & \text{if } x = 1 \\ 2 + x & \text{if } x > 1 \end{cases}$$

Determine whether there exists a value of a such that $f(x)$ is continuous at $x = 1$. Justify your answer using limits and the definition of continuity and/or determine the type of discontinuity.

END OF TEST