1. The following are unrelated (no justification is needed for this problem):

   (a) Write in interval notation: $x \le -\pi$ or $1 < x \le \sqrt{3}$

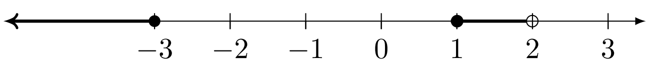
   Solution: $\boxed{(-\infty, -\pi] \cup \left(1, \sqrt{3}\right]}$.

   (b) Write in interval notation: x is less than $\dfrac{2}{5}$ and greater than or equal to zero.

   Solution: $\boxed{\left[0, \dfrac{2}{5}\right)}$.

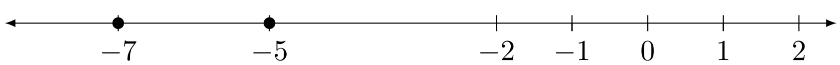
   (c) Graph on the real number line: $(-\infty, -3) \cup [1, 2)$.

   Solution:



   (d) i. Plot the numbers -5 and -7 on the real number line.

   Solution:



   ii. Is the inequality $-5 < -7$ correct based on your answer to part i?

   Solution:

   The inequality is incorrect, -7 is less than -5 based on the number line.

2. Perform the indicated operations.

   (a) $\dfrac{4}{15} - \dfrac{3}{20}$

   Solution:

$$\frac{4}{15} - \frac{3}{20} = \frac{4}{15} \cdot \frac{4}{4} - \frac{3}{20} \cdot \frac{3}{3} \tag{1}$$

$$= \frac{16}{60} - \frac{9}{60} \tag{2}$$

$$= \frac{16 - 9}{60} \tag{3}$$

$$= \boxed{\frac{7}{60}} \tag{4}$$

(b) $\frac{3}{2}\left(1+\frac{\frac{6}{5}}{3}\right)$

Solution:

$$\frac{3}{2}\left(1+\frac{\frac{5}{6}}{3}\right) = \frac{3}{2}\left(1+\frac{\frac{6}{5}}{\frac{3}{1}}\right) \tag{5}$$

$$= \frac{3}{2}\left(1+\frac{6}{5}\cdot\frac{1}{3}\right) \tag{6}$$

$$= \frac{3}{2}\left(1+\frac{2}{5}\right) \tag{7}$$

$$= \frac{3}{2}\left(\frac{5}{5}+\frac{2}{5}\right) \tag{8}$$

$$= \frac{3}{2}\left(\frac{7}{5}\right) \tag{9}$$

$$= \boxed{\frac{21}{10}} \tag{10}$$

3. Consider the list of real numbers: $\left\{\frac{1}{4}, \pi, 0, -5, 97, \sqrt{3}\right\}$

(a) Write down the irrational numbers **note that some numbers may appear in more than one part**.

Solution:

$\boxed{\pi, \sqrt{3}}$

(b) Write down the rational numbers **note that some numbers may appear in more than one part**.

Solution:

$\boxed{\frac{1}{4}, 0, -5, 97}$

(c) Write down the integers **note that some numbers may appear in more than one part**.

Solution:

$\boxed{0, -5, 97}$

4. Express the quantity without using absolute value:

(a) $|7-x|$ when $x < -7$

Solution:

$|7-x| = \boxed{7-x}$ since $7-x > 0$ for values of x that are less than -7.

(b) $|3-\pi|$

Solution:

$|3-\pi| = \boxed{-(3-\pi)}$ since $3 < \pi$ and so $3-\pi$ is negative.

5. The following are unrelated:

(a) Add as indicated: $-2^2 + 8^0 + 3^{-1}$

Solution:

$$
\begin{aligned}
-2^2 + 8^0 + 3^{-1} &= -4 + 1 + \frac{1}{3} && (11)\\
&= -3 + \frac{1}{3} && (12)\\
&= -\frac{9}{3} + \frac{1}{3} && (13)\\
&= \boxed{-\frac{8}{3}} && (14)
\end{aligned}
$$

(b) Evaluate the expression: $\dfrac{2-|4-11|}{|-1|-5}$

Solution:

$$
\begin{aligned}
\frac{2-|4-11|}{|-1|-5} &= \frac{2-|-7|}{1-5} && (15)\\
&= \frac{2-7}{-4} && (16)\\
&= \boxed{\frac{5}{4}} && (17)
\end{aligned}
$$

(c) Evaluate the expression: $\dfrac{\sqrt{32}}{\sqrt{18}}$

Solution:

$$
\begin{aligned}
\frac{\sqrt{32}}{\sqrt{18}} &= \frac{\sqrt{16\cdot 2}}{\sqrt{9\cdot 2}} && (18)\\
&= \frac{4\sqrt{2}}{3\sqrt{2}} && (19)\\
&= \boxed{\frac{4}{3}} && (20)
\end{aligned}
$$

6. The following are unrelated:

(a) Simplify the expression: $(3x-5)^2+2x^5+3-\left(2x^2x^3\right)-x$.

Solution:

$$\begin{aligned}(3x-5)^2+2x^5+3-\left(2x^2x^3\right)-x &= (3x-5)(3x-5)+2x^5+3-2x^5-x && (21)\\ &= 9x^2-15x-15x+25+3-x && (22)\\ &= 9x^2-30x+28-x && (23)\\ &= \boxed{9x^2-31x+28} && (24)\end{aligned}$$

(b) Combine into a single fraction:

$$\sqrt{4+x^2}-\frac{1}{\sqrt{4+x^2}}$$

Solution:

$$\begin{aligned}\sqrt{4+x^2}-\frac{1}{\sqrt{4+x^2}} &= \frac{\sqrt{4+x^2}}{1}-\frac{1}{\sqrt{4+x^2}} && (25)\\ &= \frac{\sqrt{4+x^2}}{\sqrt{4+x^2}}\cdot\frac{\sqrt{4+x^2}}{1}-\frac{1}{\sqrt{4+x^2}} && (26)\\ &= \frac{4+x^2}{\sqrt{4+x^2}}-\frac{1}{\sqrt{4+x^2}} && (27)\\ &= \frac{4+x^2-1}{\sqrt{4+x^2}} && (28)\\ &= \boxed{\frac{3+x^2}{\sqrt{4+x^2}}} && (29)\end{aligned}$$

7. Simplify the expression (give your answer without negative exponents) (4 pts): $3\left(a^{-1/2}\right)^{-12}+\frac{10a^8}{5a^2b^3b^{-3}}$

Solution:

$$\begin{aligned}3\left(a^{-1/2}\right)^{-12}+\frac{2a^8}{a^2b^3b^{-3}} &= 3a^6+\frac{2a^8b^3}{a^2b^3} && (30)\\ &= 3a^6+2a^6 && (31)\\ &= \boxed{5a^6} && (32)\end{aligned}$$

8. Is $x=2$ a solution of $\left(x^2+14\right)(x-3)^9(x-1)^7\left(\frac{1}{3}\right)=-6$? As usual, make sure to justify your answer, an answer without work will receive no credit.

Solution:

We start by plugging in $x=2$ into the left side of the equation.

$$\begin{aligned}\left(2^2+14\right)(2-3)^9(2-1)^7\left(\frac{1}{3}\right) &= (4+14)(-1)^9(1)^7\left(\frac{1}{3}\right) && (33)\\ &= (18)(-1)(1)\left(\frac{1}{3}\right) && (34)\\ &= (-18)\left(\frac{1}{3}\right) && (35)\\ &= -6 && (36)\end{aligned}$$

Yes $x=2$ is a solution with correct work.

9. Consider the difference of two cubes formula, $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ and the expression $8x^3 - 27y^3$.

(a) In order to apply the formula to factor $8x^3 - 27y^3$ we need to identify what a and b are in the formula. Write down the appropriate values for a and b that allow us to factor $8x^3 - 27y^3$.

Solution:

$$8x^3 = (2x)^3 \tag{37}$$
$$27y^3 = (3y)^3 \tag{38}$$
$$\boxed{a = 2x, b = 3y} \tag{39}$$

(b) Now use the formula and your answer to part (a) to factor $8x^3 - 27y^3$.

Solution:

Substitute $2x$ for a, and $3y$ for b: (40)

$$\boxed{(2x-3y)(4x^2+6xy+9y^2)} \tag{41}$$

10. Factor completely (If not factorable write NF): $2x^2y - 12xy^2 + 32xy$

Solution:

$$= \boxed{2xy(x-6y+16)} \tag{42}$$

11. Factor each of the following completely (If not factorable write NF):

(a) $x^{3/2} - 9x^{-1/2}$ (start by factoring out lowest power of x)

(b) $x^2 + 16$

Solution a):

$$= x^{-1/2}(x^2-9) \tag{43}$$
$$= x^{-1/2}(x+3)(x-3) \tag{44}$$
$$= \boxed{\frac{(x+3)(x-3)}{\sqrt{x}}} \tag{45}$$

Solution b): A sum of two squares is not factorable in the real numbers. $\boxed{\text{NF}}$

12. Simplify the complex fraction: $\dfrac{\frac{1}{x} - \frac{4}{x^3}}{1 - \frac{1}{x-1}}$

Solution:

$$= \frac{\frac{x^2}{x^3} - \frac{4}{x^3}}{1 - \frac{1}{x-1}} = \frac{\frac{x^2-4}{x^3}}{1 - \frac{1}{x-1}} = \frac{\frac{x^2-4}{x^3}}{\frac{(x-1)}{(x-1)} - \frac{1}{x-1}} = \frac{\frac{x^2-4}{x^3}}{\frac{(x-1)-1}{(x-1)}} \tag{46}$$
$$= \frac{x^2-4}{x^3} \cdot \frac{(x-1)}{(x-2)} \tag{47}$$
$$= \frac{(x+2)(x-2)}{x^3} \cdot \frac{(x-1)}{(x-2)} \tag{48}$$
$$= \frac{(x+2)\cancel{(x-2)}}{x^3} \cdot \frac{(x-1)}{\cancel{(x-2)}} \tag{49}$$
$$= \boxed{\frac{(x+2)(x-1)}{x^3}} \tag{50}$$

13. Divide: $\frac{\frac{x^3-x^2}{x^2+2x+1}}{\frac{38x-38}{4(x-1)}}$

Solution:

$$= \frac{x^3 - x^2}{x^2+2x+1} \cdot \frac{4(x-1)}{38x-38} \tag{51}$$
$$= \frac{x^2(x-1)}{(x+1)^2} \cdot \frac{4(x-1)}{38(x-1)} \tag{52}$$
$$= \frac{x^2(x-1)}{(x+1)^2} \cdot \frac{4\cancel{(x-1)}}{38\cancel{(x-1)}} \tag{53}$$
$$= \frac{4x^2(x-1)}{38(x+1)^2} \tag{54}$$
$$= \boxed{\frac{2x^2(x-1)}{19(x+1)^2}} \tag{55}$$

14. Solve each of the following equations:

(a) $x^2 + 10 = 7x$

(b) $\frac{x}{8} = \frac{3}{4}x - 3 - \frac{1}{4}$

Solution a):

$$x^2 + 10 = 7x \tag{56}$$
$$x^2 - 7x + 10 = 0 \tag{57}$$
$$(x-2)(x-5) = 0 \tag{58}$$
$$x = \boxed{2, 5} \tag{59}$$

Solution b):

$$\frac{x}{8} = \frac{3}{4}x - 3 - \frac{1}{4} \tag{60}$$
$$x = 6x - 24 - 2 \tag{61}$$
$$x - 6x = -26 \tag{62}$$
$$-5x = -26 \tag{63}$$
$$x = \boxed{\frac{26}{5}} \tag{64}$$

15. Solve each of the following equations:

(a) $\sqrt{2} = (x+3)^2$

(b) Solve for r: $2r + 3pr = -5 + r$

Solution a):

$$\sqrt{(x+3)^2} = \pm\sqrt{\sqrt{2}} \tag{65}$$
$$x + 3 = \pm\sqrt[4]{2} \tag{66}$$
$$x = \boxed{-3 \pm \sqrt[4]{2}} \tag{67}$$

Solution b):

$$2r + 3pr - r = -5 \tag{68}$$
$$r + 3pr = -5 \tag{69}$$
$$r(1+3p) = -5 \tag{70}$$
$$r = \boxed{\frac{-5}{1+3p}} \tag{71}$$

(c) Solve: $4 - 2x = x^2$

Solution:

$$x^2 + 2x - 4 = 0 \tag{72}$$

Doesn't factor nicely, use quadratic formula: (73)

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2} \tag{74}$$
$$x = \frac{-2 \pm \sqrt{4+16}}{2} \tag{75}$$
$$x = \frac{-2 \pm \sqrt{20}}{2} \tag{76}$$
$$x = \frac{-2 \pm 2\sqrt{5}}{2} \tag{77}$$
$$x = \boxed{-1 \pm \sqrt{5}} \tag{78}$$