

**APPM 1345****Exam 1****Spring 2026****Name****Instructor** Richard McNamara**Section 150**

This exam is worth 100 points and has **5 problems**.

**Make sure all of your work is written in the blank spaces provided.** If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

**Show all work and simplify your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

**End-of-Exam Procedure**

1. Go to the designated area to scan and upload your exam to Gradescope.
2. Verify that your exam has been correctly uploaded and all problems have been labeled.
3. Hand the physical copy of your exam to a proctor.
4. Have a proctor swipe your BuffOne card.

1. (26 pts) Parts (a) and (b) are not related.

(a) Let  $f(x) = \frac{3}{8}x^{8/3} - \frac{18}{5}x^{5/3} + 12x^{2/3}$ . Clearly state “none” if/where applicable.

i. Identify all critical numbers of  $f(x)$ , if any.

ii. Identify all intervals, if any, on which  $f$  is increasing and all intervals, if any, on which  $f$  is decreasing.

Intervals on which  $f$  is increasing:

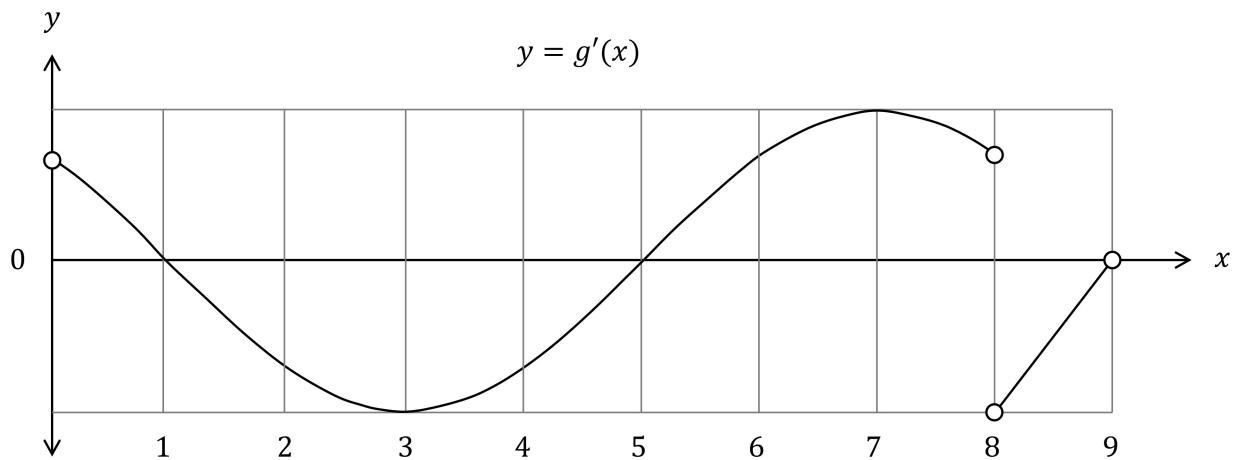
Intervals on which  $f$  is decreasing:

iii. Find the  $x$ -coordinate of every local maximum and every local minimum of  $f$ , if any. Briefly justify each answer using the First Derivative Test.

$x$ -coordinates of local maxima:

$x$ -coordinates of local minima:

(b) The graph of the first derivative  $g'(x)$  of a continuous function  $g(x)$  is shown below on the interval  $(0, 9)$ .



Answer the following questions about the continuous function  $g(x)$ . Clearly state “none” if/where applicable.

To be clear, these questions relate to  $g(x)$ , which is not the function that is graphed above.

- i. Identify all critical number(s) of  $g(x)$ , if any.
- ii. Identify all interval(s), if any, on which  $g(x)$  is increasing.
- iii. Identify the  $x$ -coordinate of every local maximum of  $g(x)$ , if any.
- iv. Identify all interval(s), if any, on which  $g(x)$  is concave down.
- v. Identify the  $x$ -coordinate of every inflection point of  $g(x)$ , if any.

2. (17 pts) Find the coordinates of the point on the curve  $y = x^2$  with  $x > 0$  that is closest to the point  $(0, 2)$ . Use the Second Derivative Test to verify that the corresponding distance is a local minimum value.

3. (16 pts) Find an equation of the slant asymptote of the function  $y = \frac{3x^4 - 4x^3 + x^2 + x - 1}{x^3 - 2x^2 + x - 1}$ .

4. (20 pts) Suppose the first and second derivatives of a continuous function  $h(x)$  on the interval  $[0, 5]$  are as follows:

$$h'(x) = (x - 1)(x - 4)^3$$

$$h''(x) = (x - 4)^2(4x - 7)$$

For each of the following questions, clearly state “none” if/where applicable.

(a) Identify all critical numbers of  $h$  on the interval  $[0, 5]$ .

(b) Identify all intervals, if any, on which  $h$  is increasing and all intervals, if any, on which  $h$  is decreasing on the interval  $[0, 5]$ .

Subintervals of  $[0, 5]$  on which  $h$  is increasing:

Subintervals of  $[0, 5]$  on which  $h$  is decreasing:

(c) Find the  $x$ -coordinate of every local maximum and every local minimum of  $h$ , if any, on the interval  $[0, 5]$ .  
Briefly justify each answer using the First Derivative Test.

$x$ -coordinates of local maxima on the interval  $[0, 5]$ :

$x$ -coordinates of local minima on the interval  $[0, 5]$ :

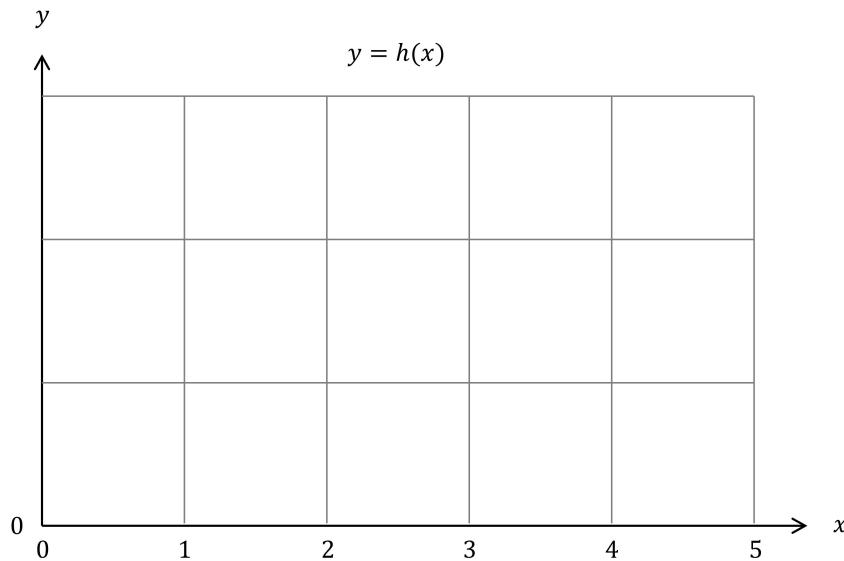
(d) Identify all intervals, if any, on which  $h$  is concave up and all intervals, if any, on which  $h$  is concave down on the interval  $[0, 5]$ .

i. Subintervals of  $[0, 5]$  on which  $h$  is concave up:

ii. Subintervals of  $[0, 5]$  on which  $h$  is concave down:

(e) Find the  $x$ -coordinate of every inflection point of  $h$  on  $[0, 5]$ , if any.

(f) Sketch  $y = h(x)$  on  $[0, 5]$  on the axis system below, assuming that  $h(0) = 0$  and  $h(x) > 0$  on  $(0, 5]$ . Your sketch should clearly indicate the  $x$ -coordinate of each local maximum, local minimum, and inflection point. The  $y$ -coordinates of those points are unimportant, although the behavior of the function curve should be clearly consistent with your answers to parts (a) - (e).



5. (21 pts) A rectangular box with a square base and a closed top is to have a volume of  $30 \text{ m}^3$ . Material for the base and the top costs \$3 per square meter and material for the sides costs \$2 per square meter. Find the length of the base of the box that minimizes the total material cost of the box, including the correct unit of measurement. Use the Second Derivative Test to verify that the result is a local minimum value.

END OF EXAM

Your Initials \_\_\_\_\_

**ADDITIONAL BLANK SPACE**

If you write a solution here, please clearly indicate the problem number.