Write your name and your professor's name or your section number below. You are *not* allowed to use textbooks, or a calculator. You may have two 3x5" cards (both sides) for notes, or equivalent. To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise.

Name:

Instructor and Section: Mitchell, 002

1. (30 pts) If the statement is **always true**, write "TRUE"; if it is possible for the statement to be false then mark "FALSE." You must give a **justification** for your answer. That is, if the answer is true, provide a brief proof. If the answer is false, provide a counterexample.

   (a) A singular matrix is not diagonalizable.

   (b) If the incomplete matrix A has a Jordan Canonical Form (JCF) J, then the JCF of A^2 is J^2.

   (c) The quadratic function $p(x, y) = 2x^2 - 2xy + y^2 - 2x + 4y + 7$ has a minimum value.

   (d) If a matrix is not symmetric, then it is incomplete.

   (e) If a system of equations $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions then the matrix A is singular.

2. (20 pts) The following questions are unrelated.

   (a) (8 pts) Is $L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ 0 \\ -x \end{pmatrix}$ a linear function? If it is linear, prove that it is and find the matrix representation of L in the standard basis.

   (b) (12 pts) Let L be the linear function with the standard basis representation given by $A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \end{pmatrix}$. Find bases for $\mathbb{R}^2$ and $\mathbb{R}^3$ that puts L into canonical form after a change of basis.

3. (20 pts) Let $A = \begin{pmatrix} -1 & 2 \\ -4 & 5 \end{pmatrix}$.

   (a) (8 pts) What is the characteristic polynomial $p(\lambda)$ for A?

   (b) (8 pts) Find the eigenvalues and eigenvectors for A.

   (c) (4 pts) Is A a complete matrix? What is the diagonalization transformation for A? (The Λ matrix in $S\Lambda S^{-1}$)

4. (20 pts) Let A be a matrix with characteristic polynomial given by

$$p_A(\lambda) = (1-\lambda)^3(2-\lambda)^3(-3-\lambda)$$

Eigenvalue $\lambda = 1$ has one ordinary eigenvector while eigenvalue $\lambda = 2$ has two ordinary eigenvectors.

(a) (16 pts) Write down all of the Jordan blocks that appear in A's Jordan Canonical Form.

(b) (4 pts) What is the dimension the of eigenspace for eigenvalue 2?

5. (20 pts) Let A be the matrix with the SVD given by

$$A = \begin{pmatrix} -1/\sqrt{3} & 1/\sqrt{7} \\ 1/\sqrt{3} & 2/\sqrt{7} \\ 1/\sqrt{3} & -1/\sqrt{7} \\ 0 & 1/\sqrt{7} \end{pmatrix} \begin{pmatrix} 4\sqrt{15} & 0 \\ 0 & \sqrt{35} \end{pmatrix} \begin{pmatrix} -1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix}$$

   (a) (2 pts) What is the rank of A? Be sure to justify your answer.

   (b) (6 pts) Does $A\mathbf{x} = \mathbf{b}$ have a solution when $\mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$?

   (c) (12 pts) What is the best rank 1 approximation of A using the Froebenius norm?

6. (20 pts) Let B be the matrix with the SVD given by

$$B = \begin{pmatrix} -1/2 & -1/2 \\ 1/2 & -1/2 \\ 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   (a) (8 pts) Find the psuedoinverse of B.

   (b) (4 pts) Find the least squares solution to $B\mathbf{x} = \mathbf{c}$ when $\mathbf{c} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$.

   (c) (8 pts) What is the closest point to $\mathbf{c}$ in $\text{img}B$?

7. (20 points) The 3×3 matrix A has eigenvalues $\lambda_1 = 2$, $\lambda_2 = 3$ and $\lambda_3 = 4$ with corresponding eigenvectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, and $\mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, respectively.

   (a) (4 pts) Does this uniquely define the matrix A? Explain your answer.

   (b) (8 pts) Find a matrix A corresponding to the given eigenvalues/eigenvectors (either "a" matrix, or "the" matrix, depending on your answer to the previous question).

   (c) (8 pts) Find e^{tA}.