

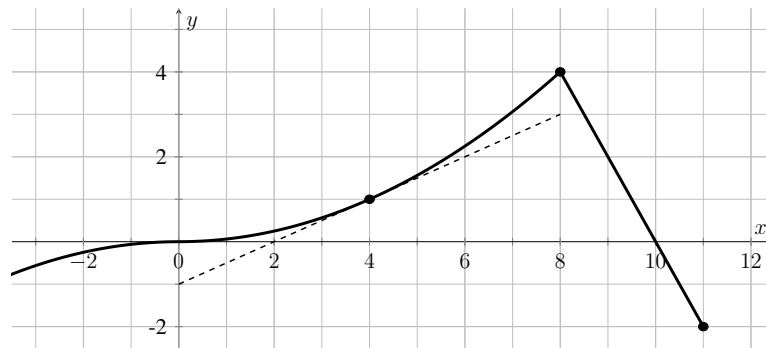
1. (27 points) The following problems are unrelated.

(a) Find the derivative of $f(x) = 7x^2 \cos(\tan x)$.

(b) Evaluate $\int \frac{x^2}{(x^3 + 4)^2} dx$.

(c) Evaluate $\lim_{x \rightarrow 0} \frac{\arcsin(2x)}{5x}$.

2. (32 points) Consider the odd function $f(x)$ defined on $[-11, 11]$. A portion of its graph is shown below. The dashed line corresponds to the line tangent to $f(x)$ at $x = 4$.



(a) Evaluate $\int_{-8}^{11} f(x) dx$.

(b) Find the linearization of $f(x)$ at $a = -4$.

(c) Let $g(x) = \int_{-11}^x f(t) dt$. Find $g'(9)$.

(d) Evaluate $\lim_{x \rightarrow 4} \frac{f(x) - 1}{6x - 24}$.

3. (13 points) In the year 1990, the population of Tralfamador was 5 billion. By the year 2010, the population was 6 billion. Assuming the growth of the population is proportional to the current size of the population, that is $\frac{dP}{dt} = kP$, what will the population be in the year 2042?

4. (18 points) The graph of a function $f(x)$ is shown below. Suppose $f(x)$ is the **derivative** of $F(x)$. That is, $f(x) = F'(x)$. Assume that $F(x)$ is continuous on the interval $[m, r]$. No justification is required for the following questions. If the answer to any question is “none”, write “none”.

(a) On what open interval(s) is F decreasing?

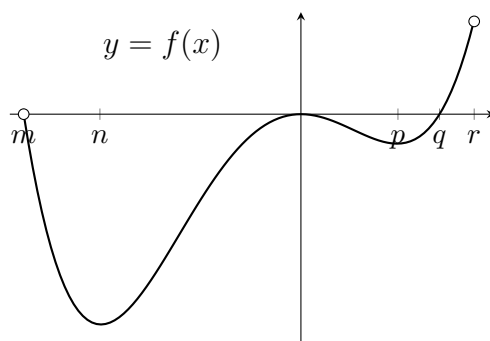
(b) On what open interval(s) is F concave down?

(c) What is the x -coordinate of the absolute maximum value of F ?

(d) What is the x -coordinate of the absolute minimum value of F ?

(e) What are the x -coordinates of the inflection point(s) of F ?

(f) At what value(s) of x does $\lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h}$ equal 0?



5. (19 points) The following problems are unrelated.

(a) Find the derivative of $y = (x^2 + 9x^4)^{\sin x}$. Your final answer should be in terms of x , but otherwise unsimplified.

(b) Consider the integral $\int_1^2 \frac{1}{x} dx$.

- Approximate this integral using a left Riemann sum (use lefthand endpoints) with three rectangles of equal width. Fully simplify your final answer.
- Is your approximation from (i) an overestimate or underestimate? Provide a *brief* justification for your answer.

6. (27 points) Consider $h(x) = \frac{e^x}{1 + e^{2x}}$. This function will be used throughout this problem.

(a) Determine $h(\ln 2)$. Be sure to fully simplify your final answer.

(b) Evaluate $\int_0^{\ln \sqrt{3}} h(x) dx$. Be sure to fully simplify your final answer.

(c) Use the definition of an even function to show that $h(x)$ is an even function.

7. (14 points) Irwin is building a cage for his pet snake. The cage will have a volume of 6 cubic feet and a square base measuring x by x feet. Material for the cage top and bottom will cost \$2 per square foot, and material for the glass sides will cost \$3 per square foot. What value of x will minimize the cost of materials? Use the Second Derivative Test to confirm that x produces a minimum.

