

**APPM 1340**  
**Final Exam**  
**Fall 2025**

<b>Name</b>
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This exam is worth 150 points and has **7 problems**.

**Make sure all of your work is written in the blank spaces provided.** If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

**Show all work and simplify your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

#### **End-of-Exam Procedure**

1. Go to the designated area to scan and upload your exam to Gradescope.
2. Verify that your exam has been correctly uploaded and all problems have been labeled.
3. Hand the physical copy of your exam to a proctor.
4. Have a proctor swipe your BuffOne card.

#### **Potentially Useful Formulas**

##### **Sums and differences:**

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta\end{aligned}$$

##### **Double-angle formulas:**

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta\end{aligned}$$

1. (26 pts) The position function of a particle is given by  $s(t) = -t^3 + 3t$  on the interval  $[0, 3]$ , where  $t$  is measured in seconds and  $s$  is measured in meters.

(a) i. Find all critical numbers of  $s(t)$  on the interval  $[0, 3]$ . If none exist, clearly state “none”.

ii. Identify the absolute maximum and minimum values of  $s(t)$  on the given interval and the corresponding values of  $t$  at which they occur.

- (b) i. Determine the total distance traveled by the particle between  $t = 0$  and  $t = 3$  seconds. Include the correct unit of measurement.

- ii. Determine the particle's acceleration at  $t = 2$  seconds. Include the correct unit of measurement.

2. (21 pts) Consider the following relationship:  $x \cos y = 3y + 4x^3$ .

(a) Find an expression for  $\frac{dy}{dx}$ .

(b) Find an equation of the tangent line to the curve  $x \cos y = 3y + 4x^3$  at the point  $(1/2, 0)$ .

3. (20 pts) Consider two fixed points such that point A is 3 miles west of point B.

Suppose a man starts walking north from point B at a constant speed of 4 miles per hour at the same time a woman starts walking west from point A at a constant rate of 2 miles per hour.

How fast is the distance between the two people increasing when the man is 4 miles north of point B? Include the correct unit of measurement.

4. (23 pts) Parts (a) and (b) are unrelated.

(a) i. Find the linearization of  $f(x) = \sqrt{1+x}$  centered at  $x = 24$ .

ii. Use the linear approximation from part (a) to estimate the value of  $\sqrt{26}$ .

(b) The radius of a circle was measured and found to be 20 cm with a possible error in measurement of at most 0.1 cm. Suppose that value of the radius is used to compute the area of the circle. Use differentials to determine the following values. Include the correct unit of measurement where applicable.

- i. The maximum possible error in the computed value of the area.
- ii. The maximum possible relative error in the computed value of the area.

5. (24 pts) Parts (a) and (b) are not related.

(a) Let  $g(x) = x^{1/2} - x^{3/2}$  on the interval  $[0, 1]$ .

i. Verify that  $g(x)$  satisfies all three hypotheses of Rolle's Theorem on the specified interval.

ii. Find all numbers  $c$  that satisfy the conclusion of Rolle's Theorem.

(b) Let  $h(x) = x^3 - 3x^2 - x$  on the interval  $[1, 4]$ .

i. Verify that  $h(x)$  satisfies both hypotheses of the Mean Value Theorem on the specified interval.

ii. Find all numbers  $c$  that satisfy the conclusion of the Mean Value Theorem.

6. (15 pts) Consider the rational function  $r(x) = \frac{x^2 + 2x + 1}{1 - x^2}$ .

- (a) Find all values of  $x$  corresponding to removable discontinuities of  $r(x)$ , if any exist. If no removable discontinuities exist, clearly state “none”. Support your answer by evaluating the appropriate limit(s).
- (b) Find the equation of every vertical asymptote of  $y = r(x)$ , if any exist. If no vertical asymptotes exist, clearly state “none”. Support your answer by evaluating the appropriate limit(s).

7. (21 pts) Parts (a) and (b) are unrelated.

- (a) Determine  $u'(x)$  for the function  $u(x) = 5x - x^2$  by using the definition of derivative. You must obtain  $u'$  by evaluating an appropriate limit to earn credit.

(b) Consider the function  $w(x)$ , which is defined as follows:

$$w(x) = \begin{cases} \frac{\sin(3x)}{x} & , \quad x < 0 \\ \frac{x^2 - 2x - 8}{x - 4} & , \quad x > 0 \end{cases}$$

- i. Evaluate both one-sided limits of  $w(x)$  as  $x$  approaches zero.
- ii. Briefly explain why  $w(x)$  has a jump discontinuity at  $x = 0$ .

END OF EXAM

Your Initials \_\_\_\_\_

**ADDITIONAL BLANK SPACE**

If you write a solution here, please clearly indicate the problem number.