APPM 1235 **Final Exam** **Fall 2025**

INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no/insufficient justification may receive no/limited credit. **Not allowed:** Calculators, notes, phones, electronic earphones, electronic devices, other unauthorized devices, and getting help from (or giving help to) another person while taking the exam. The exam is worth 150 points.

Potentially useful formulas:

1. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

NOTE: TEAR THIS PAGE OFF AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE

iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND MOVE TO THE EXAM SUBMISSION AREA. SUBMIT YOUR EXAM TO GRADESCOPE AND TURN IN THE PAPER COPY TO THE PROCTOR.

v. IF THE UPLOAD AREA IS VERY CROWDED, WE RECOMMEND CHECKING YOUR WORK AND WAITING UNTIL THE AREA IS LESS BUSY. THANK YOU FOR YOUR PATIENCE!

Scratch Paper

Name: _____

1. The following are unrelated. (7 pts)

   (a) Perform the indicated operations: $\dfrac{5}{18} + \dfrac{1}{12} - 2^{-1}$

   (b) Evaluate the expression: $-\dfrac{\sqrt{18}}{2\sqrt{32}}$

2. Rewrite each of the following without absolute value symbol (4 pts):

   (a) $|z + 5|$ where $z < -5$

   (b) $|e - 1|$

3. Let x and y be real numbers such that $x \geq 0$ and $y < 0$. Determine whether the following expression is positive, negative, or the sign cannot be determined (2 pts):

$$-x^2y^3 - 2y$$

4. The following are unrelated. (15 pts)

(a) Perform the indicated operations: $5x(2x - x^3) + (2x^2 - 3)^2$

(b) Perform the indicated operations (leave answer without negative exponents):

$$(-xy^4)^2 \left(\frac{xy^{-3}}{2}\right) + 2\frac{x^{\frac{9}{2}}}{\sqrt{x}}$$

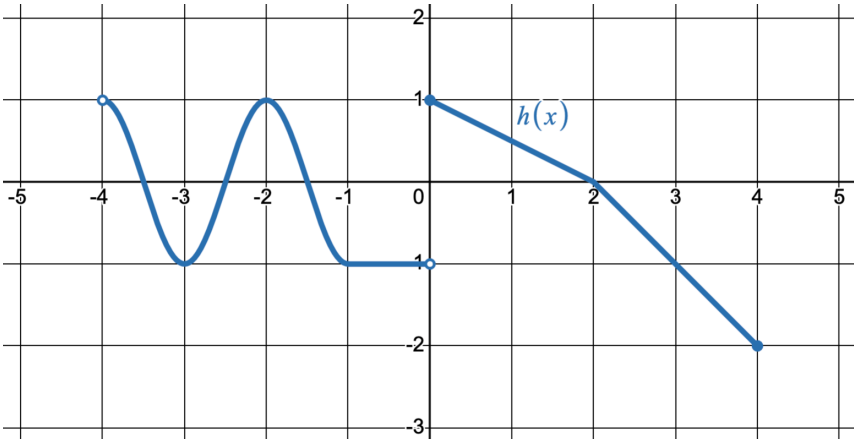
(c) Perform the indicated operation: $\dfrac{x}{2x - 26} - \dfrac{1}{x^2 - 13x}$

5. The following are unrelated. (9 pts)

(a) Perform the indicated operations: $\left(2^{-x}+3\cdot 2^{3x}\right)2^{3x}+\left(2^{x}\right)^{6}$

(b) Evaluate the following: $10^{2\log(3)}+\log_3\left(3^{x-1}\right)-\ln(1)$ (Your answer should have no logarithms)

6. Use the graph of $h(x)$ below to answer the following (11 pts):



(a) Identify the domain of $h(x)$.

(b) Identify the range of $h(x)$.

(c) Solve the equation $h(x)=1$.

(d) If $g(x)=x^2-\sin(x+1)$ find $g(h(3))$

(e) $h(x)$ is not currently a one-to-one function. Identify a restriction of the domain that preserves the range and results in a one-to-one function. Give answer in interval notation.

7. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (12 pts)

(a) Solve for x: $\quad 4 - x^2 = 5x$

(b) Solve for k: $\quad k = \frac{1}{2} n\,(1 - k)$

(c) Solve for r: $\quad 3e^{r-1} + 4 = 2$

8. A chemist measures the volume, V, of a certain substance and also its temperature, t. Volume has units of centimeters cubed and temperature has units of degrees Celsius. The volume of the substance is 8 centimeters cubed when its temperature is 5 degrees Celsius and the volume is 11 centimeters cubed when the temperature is 7 degrees Celsius. The chemist finds that the volume of the substance is a linear function of its temperature: $V(t) = mt + b$. Use this information to answer the following. (11 pts)

   (a) What is the value of the slope of the linear function $V(t) = mt + b$?

   (b) What are the units of the slope?

   (c) Find the equation of the linear function, $V(t) = mt + b$, for this particular substance.

   (d) Use your answer from part (c) to find the temperature of the substance when its volume is 16 centimeters cubed.

9. Consider the functions: $f(x) = \ln(x)$ and $g(x) = \dfrac{1}{x-2}$. (9 pts)

   (a) Fill in the blank for the function $f(x)$: $f(x) \to$ ____ as $x \to \infty$.

   (b) Find $(g \circ f)(x)$.

   (c) Find the domain of $(g \circ f)(x)$. Give your answer in interval notation.

10. For the rational function $R(x) = \dfrac{x^2 - 9}{x^3 + 12x^2 + 27x}$ answer the following: (10 pts)

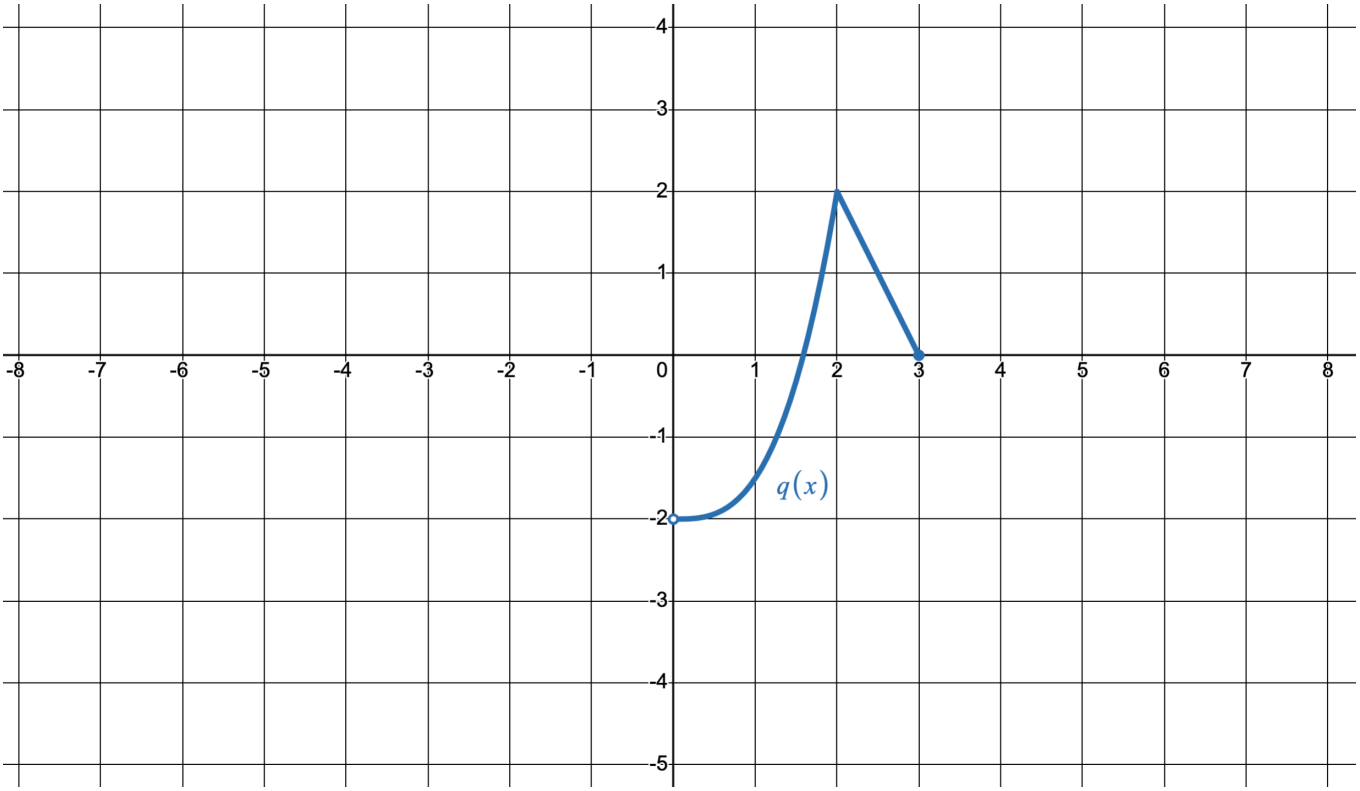
(a) Find the x-coordinate of any hole(s). If there are no hole(s) write NONE.

(b) Find the y-coordinate of any hole(s). If there are no hole(s) write NONE.

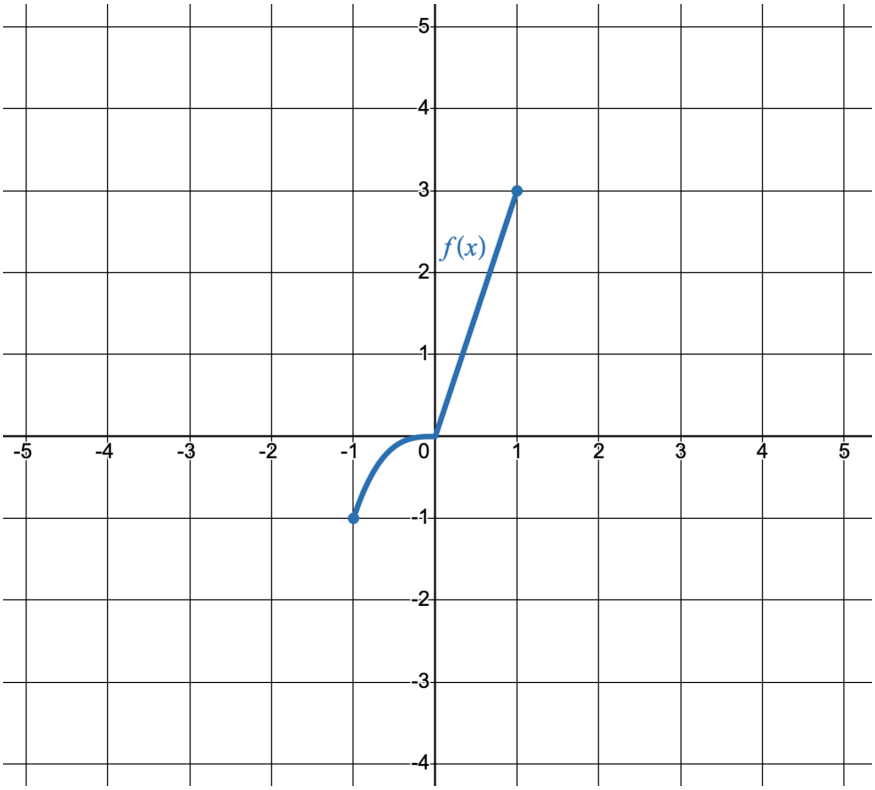
(c) Identify the horizontal or slant asymptote of $R(x)$. If there is no horizontal or slant asymptote write NONE.

(d) Find all vertical asymptote(s). If there are none write NONE.

11. For the graph of the function $q(x)$ below use transformations to sketch the graph of $q(x)+2$ **on the same set of axes**. (3 pts)

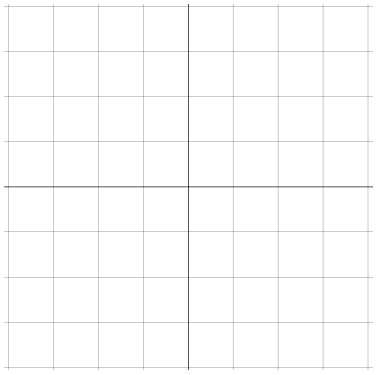


12. For the graph of $f(x)$ below, sketch the graph of $f^{-1}(x)$ **on the same set of axes below**. (4 pts)

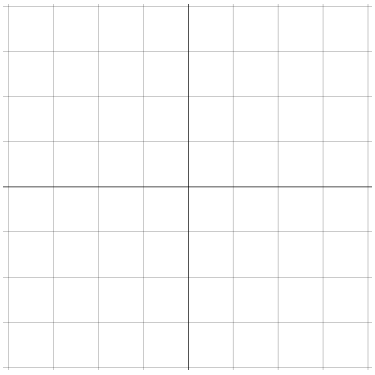


13. Sketch the graph of the following functions. **Label all intercepts and asymptotes as appropriate.** (14 pts)

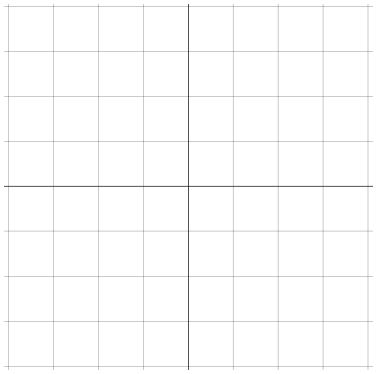
(a) $g(x) = \sqrt{x-1}$



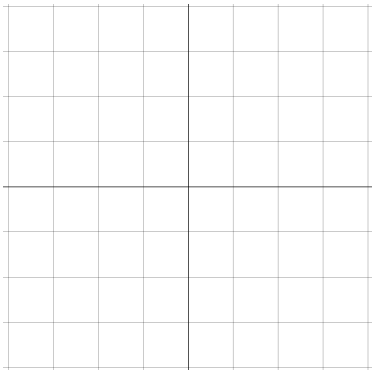
(b) $n(x) = \begin{cases} -2x-1 & \text{if} \quad -2 \le x < 0 \\ 3 & \text{if} \quad x = 0 \\ e^x & \text{if} \quad x > 0 \end{cases}$



(c) $r(x) = \sin(x)$ on the restricted domain $[0, 2\pi]$



(d) One cycle of $j(x) = \csc(x)$



14. Find the exact value for each (do not attempt to find decimal approximations): (13 pts)

(a) $\cos\left(\dfrac{5\pi}{6}\right)$

(b) $\sin^{-1}\left(\dfrac{1}{2}\right)$

(c) $\tan(225^\circ)$

(d) $\cos^{-1}\left(\cos\left(-\dfrac{\pi}{3}\right)\right)$

15. Find all solutions to the following equations: (8 pts)

(a) $2\cos(\theta) - \sqrt{2} = 0$

(b) $\tan\theta \sin\theta - \tan\theta = 0$

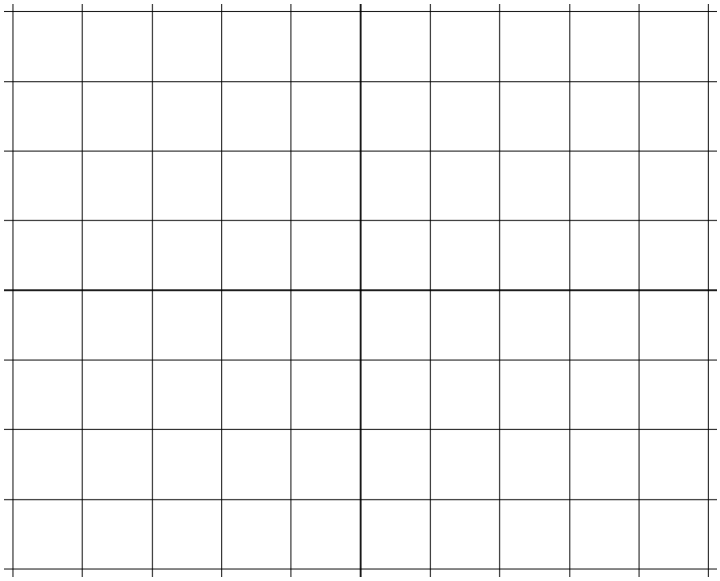
16. For $m(x) = 5\cos\left(\frac{x}{2}\right)$ (7 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $m(x)$. **Label** at least two values on the x-axis and clearly identify the amplitude.



EXAM CONTINUES ON BACK

17. Verify the identity: $\cos\alpha \cot\alpha = \csc\alpha - \sin\alpha$. (5 pts)

18. To estimate the height of a cliff above a level plain, a surveyor measures the angle of elevation to the top of the cliff to be 45°. Five hundred feet closer to the cliff along the plain, the angle of elevation is found to be 60°. Find the exact height of the cliff (do not attempt to round to a decimal number). (6 pts)

END OF FINAL EXAM