APPM 1360

Final Exam

Fall 2025

Name	
Instructor	**Section**

This exam is worth 150 points and has **7 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **match each problem with your work**

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 9:45 AM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been matched.
   - Leave the physical copy of the exam with your proctors.
2. If you finish the exam after 9:45 AM: Please wait in your seat until 10:00 AM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been matched.
   - Leave the physical copy of the exam with your proctors.

FORMULA SHEET CAN BE FOUND ON THE NEXT PAGE

Trigonometric identities

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin^2(x) = \tfrac{1}{2}\left(1 - \cos(2x)\right)$$
$$\cos^2(x) = \tfrac{1}{2}\left(1 + \cos(2x)\right)$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(u/a) + C$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(u/a) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(u/a) + C$$

Error Bounds for Trapezoidal and Midpoint Rules

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \qquad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Center of Mass Integrals

$$M = \int_a^b \rho(f(x) - g(x))\,dx$$
$$M_y = \int_a^b \rho x(f(x) - g(x))\,dx$$
$$M_x = \int_a^b \frac{1}{2}\rho\left[(f(x))^2 - (g(x))^2\right]dx$$
$$\bar{x} = \frac{M_y}{M} \text{ and } \bar{y} = \frac{M_x}{M}$$

Frequently Used Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad R = 1$$
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$
$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad R = \infty$$
$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad R = \infty$$
$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad R = 1$$
$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad R = 1$$
$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \qquad R = 1$$

Taylor Series and Taylor's Formula

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$$

Ellipse and Hyperbola

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Area

$$A = \int_a^b g(t)f'(t)\,dt \qquad x = f(t),\ \ y = g(t)$$
$$A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta$$

Volume

$$V = \int_a^b A(x)\,dx \qquad V = \int_a^b \pi\left(R^2 - r^2\right)dx$$
$$V = \int_a^b 2\pi r h\,dx$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$
$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

Surface Area

$$S = \int_a^b 2\pi r\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

1. Do each of the following converge or diverge? If it converges, find its limit. If it diverges, explain.

   (a) (8 points) $\displaystyle\sum_{n=2}^{\infty} \frac{n+4}{(n^4-2)^{1/3}}$

   (b) (18 points) $\displaystyle\int_0^{\infty} \frac{3}{(x+1)(x^2+2)}\, dx$

2. Two unrelated problems.

(a) (12 points) Let $S = \sum_{n=1}^{\infty} a_n$ and suppose the n^{th} partial sum is $s_n = \frac{5}{2}\left(1 - \frac{1}{5^n}\right)$.

i. Find an expression for a_1, a_2 and a_n.

ii. Does $S = \sum_{n=1}^{\infty} a_n$ converge? If so, find its limit. If not, explain.

(b) (18 points) Suppose $g(x) = \frac{\ln(1+x) - x}{x^2}$

i. Find the Maclaurin series for $g(x)$ (use sigma notation). Use $g(0) = -1/2$.

ii. Find the tenth derivative of g at 0, that is, find $g^{(10)}(0)$.

3. (16 points) Consider the equation $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$.

   (a) Graph this equation. Label all intercepts.

   (b) Rotate the area bounded by this equation around the line $x = 6$. Set up, but do not evaluate, an integral to find the volume of this solid.

4. (28 points) Let $f(x) = e^{-3x}$.

   (a) Find the Taylor series (use sigma notation) for f **centered at** $a = 2$.

   (b) Find the radius of convergence of the series. Justify your answer using appropriate test(s).

   (c) Find $T_2(x)$, the 2nd order Taylor polynomial.

   (d) Use Taylor's formula to find an error bound if $T_2(x)$ is used to approximate $f(x)$ for $2.5 < x < 3$.

5. (16 points) Consider the series

$$1 + x^2 + x^4 + x^6 + \cdots = \sum_{n=0}^{\infty} x^{2n},$$

for $|x| < 1$. In the following problems, your answer should not have sigma notation.

(a) Find the sum of the series.

(b) Find the sum of the series $\sum_{n=1}^{\infty} 2nx^{2n}$.

(c) Find the sum of the series $\sum_{n=1}^{\infty} \frac{2n}{3^n}$.

6. (18 points) Consider the parametric equations for one arch of the cycloid given $x = a(t - \sin(t))$ and $y = a(1 - \cos(t))$, for $0 \leq t \leq 2\pi$ and a a fixed, positive constant.

   (a) Find the area under one arch of the cycloid.

   (b) Set up, but don't evaluate, the integral to find the length of one arch of the cycloid.

7. (16 points) Consider the two circles given by $r = 1$ and $r = 2\cos(\theta)$

   (a) Graph both circles in the xy-plane. Clearly label the center of each circle and all points of intersection.

   (b) Find the area of intersection of both circles, that is, find the area common to both circles.

ADDITIONAL BLANK SPACE

If you write a solution here, please indicate the problem number and leave a note on the page where you started the problem.