1. (54 pts) Consider the surface $x^2 - 10x + 4y^2 - 4z^2 + 25 = 0$.
   (a) Points $P(11, 0, 3)$ and $Q(5, -2, 2)$ lie on the surface.
      i. Find a vector of length 4 in the direction of $\mathbf{PQ}$.
      ii. Find symmetric equations for the line passing through P and Q.
      iii. Find an equation for the plane tangent to the surface at P.
      iv. Let $h(x, y, z) = y + xz$. Find the directional derivative of h at P in the direction toward Q.
   (b) Write the equation for the surface in standard form and identify the surface.
   (c) Classify the type of curves in the $x = 0$, $y = 0$, and $z = 0$ traces of the surface.
   (d) The path $\mathbf{r}(t) = \left\langle 4 + t^2,\ t,\ \frac{1}{2} + \frac{1}{2}t^2 \right\rangle$ lies on the surface. Let the temperature along the path be $T(x, y, z) = \dfrac{y}{x} - z$. Find dT/dt, the rate at which the temperature is changing, at $t = 2$.

2. (18 pts) A joint probability density function for random variables X and Y is

$$f(x, y) = \frac{1}{2\pi} e^{\left(-x^2 - y^2\right)/2}$$

   for all real x, y. Consider the probability $P\left(Y \geq 0 \text{ and } X^2 + Y^2 \geq 16\right)$.
   (a) Set up a double integral in polar coordinates to compute the probability.
   (b) Evaluate the integral.

3. (28 pts) Region E lies in the **first octant**, below a sphere of radius 3 centered at the origin, and above the surface $z = 3 - \sqrt{x^2 + y^2}$.
   (a) Sketch and shade a cross-section of the region in the rz-plane (i.e., a half-plane of constant θ).
   (b) Set up (but do not evaluate) an integral to find the volume of the region using
      i. rectangular coordinates in the order $dz\,dy\,dx$
      ii. cylindrical coordinates in the order $dz\,dr\,d\theta$
      iii. spherical coordinates in the order $d\rho\,d\phi\,d\theta$.

4. (14 pts) Let A and B be symmetric $n \times n$ matrices. Prove that $(AB)^T = BA$ in two ways:
   (a) Using properties of matrix transpose and multiplication.
   (b) Examining the (i, j) entries of $(AB)^T$ and BA.

5. (14 pts) Use row reduction to find the inverse of the matrix.

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 1 & -2 & 0 \end{bmatrix}$$

6. (10 pts) A homogeneous system has 2 equations and 5 variables x_1, x_2, x_3, x_4, x_5. The fundamental solutions for the system are

$$b(-5, 1, 0, 0, 0) + d(2, 0, 0, 1, 0) + e(0, 0, 3, 0, 1)$$

   where $x_2 = b$, $x_4 = d$, and $x_5 = e$, for real numbers b, d, and e. Find the reduced row echelon form (RREF) augmented matrix that represents the system.

7. (12 pts) Suppose we wish to find a quadratic least-squares polynomial for the points

$$(-1,-2), \quad (0,0), \quad (1,0), \quad (2,-1).$$

Set up an augmented matrix representing the linear system that can be solved to determine the quadratic polynomial. (Do not solve the system.)