INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from (or giving help to) another person are not permitted while taking the exam. The exam is worth 100 points.

Potentially useful formulas:

(i) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(ii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iii) Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE.

iii. WRITE YOUR NAME ON THE NEXT PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND MOVE TO THE SUBMISSION AREA TO UPLOAD YOUR ANSWERS WITH SUPPORTING WORK TO GRADESCOPE AND HAND IN YOUR PHYSICAL COPY.

v. IF THE UPLOAD AREA IS VERY CROWDED, WE RECOMMEND CHECKING YOUR WORK AND WAITING UNTIL THE AREA IS LESS BUSY. THANK YOU FOR YOUR PATIENCE!

Name: _____

1. A physicist observes that the position, s measured in feet, of a certain object depends on time, t measured in seconds. The physicist establishes that the relationship is linear with equation: $s(t) = \frac{3}{2}t + 4$. (9 pts)

   (a) What are the units of the slope of the linear equation?

   (b) How far does the object move every 2 seconds?

   (c) What was the position of the object at time $t = 0$ seconds?

2. Solve the following equations. (10 pts)

(a) $\sqrt{2x} + 3 = x - 1$

(b) $\dfrac{2x}{x^2 - 4} = \dfrac{1}{2\left(x^2 - 4\right)} - \dfrac{1}{x + 2}$

3. Solve the following inequalities. Justify your answers by using a number line or sign chart when appropriate. Answers without full justification will not receive full credit. Express all answers in interval notation. (9 pts)

(a) $x^4 + 14x^3 + 33x^2 < 0$

(b) $|5x + 2| \leq 6$

4. Find the value(s) for c such that the midpoint between $(3, c)$ and $(-4, 5)$ is $\left(-\frac{1}{2}, \frac{2}{3}\right)$. (4 pts)

5. Find the domain of the following functions. Express your answers in interval notation. (15 pts)

(a) $s(x) = \dfrac{8x^2 - 4}{x^2 - 5x}$

(b) $h(x) = \dfrac{\sqrt{x-2}}{3x - 9}$

(c) $f(r) = 5r^2 - 17r + 1$

6. For $f(x) = 2 - 3x^2$ answer the following: (7 pts)

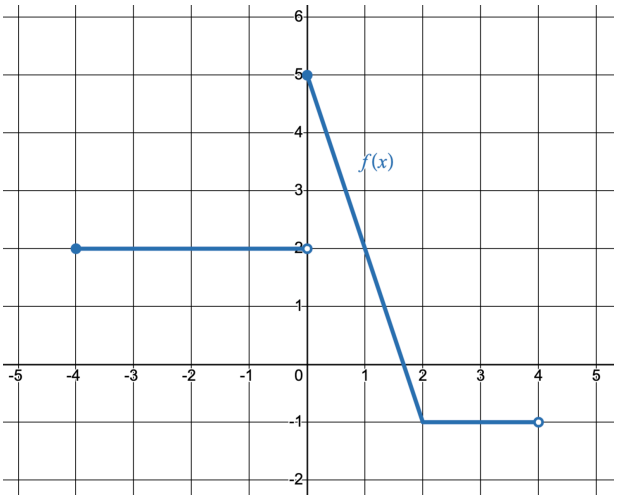
   (a) Find $f(a)$

   (b) Find $f(a + h)$

   (c) Find $\dfrac{f(a+h) - f(a)}{h}$ where h is a nonzero constant and, as usual, simplify.

7. Find the inverse function of $g(x) = 2 - 5x^3$ (you may assume that $g(x)$ is one-to-one). (4 pts)

8. Answer the following for the given graph of a function $f(x)$. (17 pts)



(a) Find the domain of $f(x)$. Express your answer in interval notation.

(b) Find the range of $f(x)$. Express your answer in interval notation.

(c) Find $f(0)$

(d) Find $(f \cdot f)(-3)$.

(e) Find $(f \circ f)(1)$.

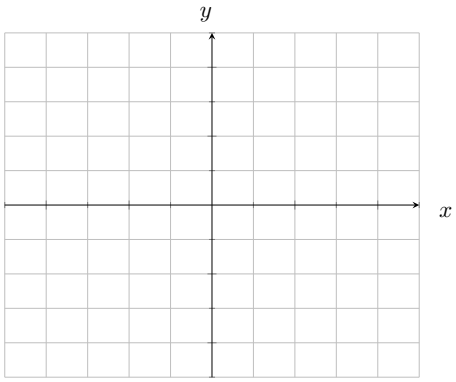
(f) $f(x)$ is **not** one-to-one. Give a brief explanation as to why $f(x)$ is not one-to-one.

(g) Find the equation of the line in the graph above on the restricted domain $[0, 2]$.

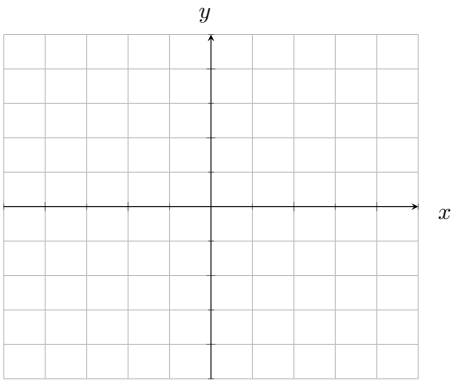
(h) Write down the piecewise-defined function whose graph is given above.

9. Sketch the shape of the graph of each of the following on the given set of axes. (17 pts)

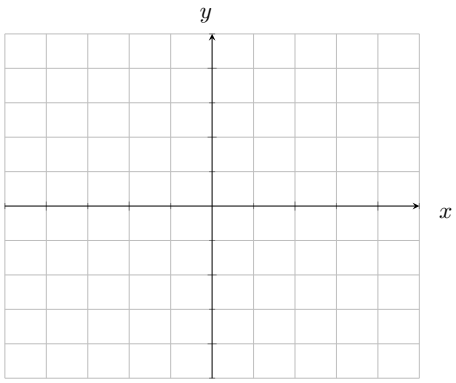
(a) $f(x) = -\sqrt{x}$



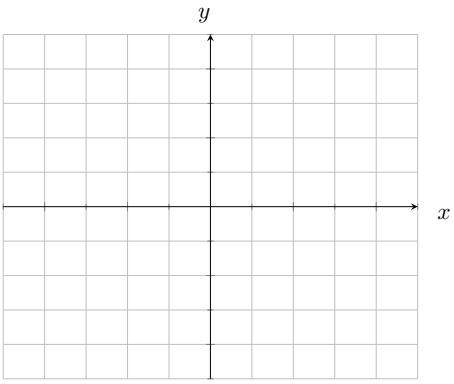
(b) $g(x) = x^3 + 2$



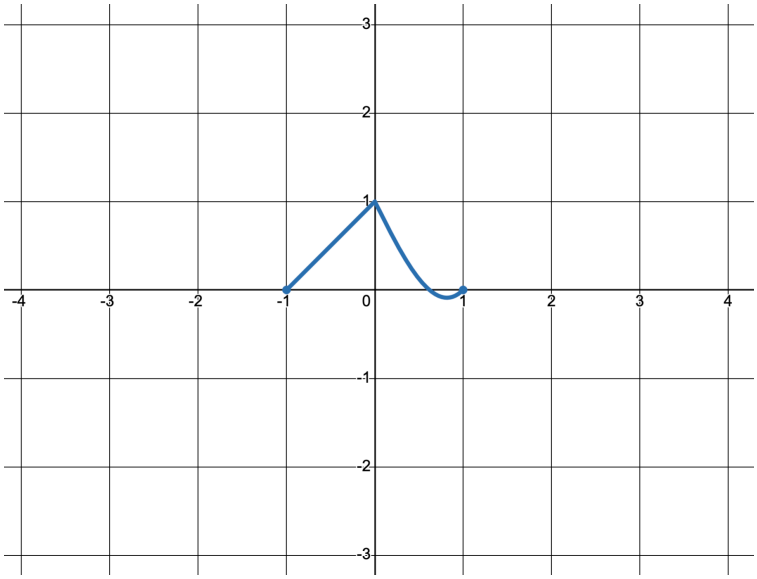
(c) $x^2 + y^2 = 4$



(d) $q(x) = \begin{cases} -2x + 3 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$



(e) For the function $h(x)$, graphed below, answer the following:



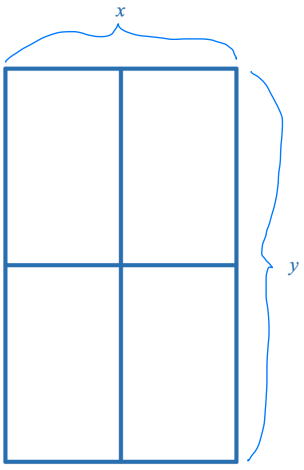
i. Use transformations to sketch the graph of $h(x) - 1$ **on the same graph above**.

ii. Use transformations to sketch the graph of $h(x + 3)$ **on the same graph above**.

EXAM CONTINUES ON BACK

10. Is $f(x) = x^4 - |x| + 3$ odd, even, or neither? Justify your answer for credit. (4 pts)

11. A farmer is fencing in a field for her horses. The entire field is the shape of a rectangle with width x and length y. She is then going to add a length of fencing to split the field through the middle and another length of fencing to create four equal sized rectangular pens (the picture below illustrates the field after all fencing has been added). The farmer has a total of 930 ft of fencing available. Answer the following. (6 pts)



(a) Find an equation for the total area of the field as a function of width, x.

(b) Use the vertex formula to find the dimensions of the field that maximize the total area of the field.