APPM 1360
Exam 3
Fall 2025

Name	
Instructor	**Section**

This exam is worth 100 points and has **4 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **match each problem with your work**

Show all work and simplify your answers. Name any theorem or test that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 8:00PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been matched.
   - Leave the physical copy of the exam with your proctors.
2. If you finish the exam after 8:00 PM: Please wait in your seat until 8:15 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been matched.
   - Leave the physical copy of the exam with your proctors.

Formulas

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Important Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

1. (30 points, 10 points each)

   Identify each of the following series as absolutely convergent, conditionally convergent, or divergent. Justify your work. As with all problems on this exam, name any test or theorem that you use.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n^2)}$

   (b) $\displaystyle\sum_{j=1}^{\infty} \frac{(3j)^j}{e^j(j+1)^j}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{k+5}{(k^7+k^2)^{1/3}}$

2. (20 points) Consider the power series $f(x) = \sum_{n=1}^{\infty} \frac{(3x-2)^n}{2^n n}$.

   (a) Determine the radius of convergence for $f(x)$.

   (b) Determine the interval of convergence for $f(x)$.

   (c) For the following questions, you may use inequalities, interval notation, or list all the values. You may also say "there are no such x-values" if there are none.

      i. For which x values is this series absolutely convergent?

      ii. For which x-values is this series conditionally convergent?

      iii. For which x-values is this series divergent?

3. (24 points) For parts (a) and (b) below, your answer should be written using sigma notation.

   (a) Find the Maclaurin series for $f(x) = \dfrac{\cos x - 1}{x}$. (Use $f(0) = 0$.)

   (b) Evaluate the indefinite integral $\displaystyle\int \frac{\cos x - 1}{x}\, dx$ as an infinite series.

   (c) Estimate the error in evaluating $\displaystyle\int_0^1 \frac{\cos x - 1}{x}\, dx$ using the first three nonzero terms of the series you found in part (a). Leave your answer in factored form.

4. The following questions are unrelated.

   (a) (10 points) Let $f(x) = (1+x)^{1/2}$. Find the second-order Taylor polynomial, $T_2(x)$, for f **centered at** $\mathbf{a = 3}$.

   (b) (6 points) Find the value of $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{\pi^{2n}}{4^{2n}(2n)!}$

   (c) (10 points) Find the value of $\displaystyle\sum_{k=1}^{\infty} \frac{2}{k(k+2)}$. (Hint: use partial fractions to first find a simple expression for s_n, the n^{th} partial sum of the series.)

ADDITIONAL BLANK SPACE

If you write a solution here, please indicate the problem number and leave a note on the intended page.