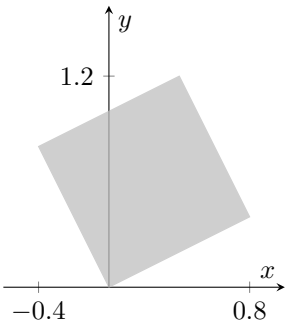
1. (16 pts) Use Lagrange multipliers to find the point(s) on the hyperbola $x^2 - y^2 = 1$ that are closest to the point $(0, 4)$. (*Hint:* You may minimize the square of the distance.)

2. (16 pts) Consider the integral $\iint_R (2x+y)^2\sqrt{x-y}\,dx\,dy$, where R, shown at right, is bounded by



$$y = \frac{x}{2},\ \ y = \frac{x}{2} + 1,\ \ y = -2x,\ \ y = -2x + 2.$$

   (a) Let $u = 2x + y$ and $v = x - y$. Sketch the transformed region in the uv-plane.

   (b) Set up (but do not evaluate) an equivalent uv-integral.

3. (18 pts) Evaluate $\displaystyle\int_0^\infty \int_0^\infty \frac{1}{(1 + x^2 + y^2)^2}\,dx\,dy$ by converting to a polar double integral.

4. (28 pts) Consider the solid with volume $V = \displaystyle\int_0^\pi \int_0^5 \int_r^5 r\,dz\,dr\,d\theta$ in cylindrical coordinates.

   (a) Sketch and shade a cross-section of the solid in the rz–plane (that is, a half-plane of constant θ). Label the intercepts.

   (b) Set up (but do not evaluate) an equivalent integral using

      i. rectangular coordinates in the order $dz\,dy\,dx$

      ii. spherical coordinates in the order $d\rho\,d\phi\,d\theta$.

5. The following two problems are not related.

   (a) (12 pts) Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = \begin{cases} k & \text{if } 0 \le x \le 50,\ 10 \le y \le 25 \\ 0 & \text{otherwise.} \end{cases}$$

      i. Find the constant k.

      ii. Set up (but do not evaluate) a double integral that represents $P\left(Y \ge \frac{X}{2}\right)$.

   (b) (10 pts) A matrix A is *skew-symmetric* if $A = -A^T$.

      i. Give an example of a 3×3 matrix B with nonzero entries that is skew-symmetric.

      ii. Compute $B - B^T$.