1. (32 points) The following problems are not related.
   (a) Suppose $g'(x) = \sin x$ and $g(\pi/4) = 3$. Determine $g(x)$.
   (b) Evaluate $\displaystyle\int_1^4 \frac{5x^2 - 3x + \sqrt{x}}{\sqrt{x}}\,dx$.
   (c) Suppose the average value of $f(x)$ along $[-1, 5]$ is 3, and that $\displaystyle\int_2^5 f(x)\,dx = 8$. Find $\displaystyle\int_{-1}^2 f(x)\,dx$.
   (d) Evaluate $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(2 + \frac{3i}{n}\right)\cdot\frac{3}{n}\right]$ using any method discussed in this class.

2. (22 points) Consider $f(x) = x^2 - 3x - 3$, which is referenced in each part of this problem.
   (a) i. Show that f satisfies the hypotheses of Rolle's Theorem on the interval $[-1, k]$ for some value of $k > -1$. What must k equal?
   ii. Determine the value(s) c that satisfy the conclusion of the Rolle's Theorem for $f(x)$ over $[-1, k]$, where k is the value you found in (i).
   (b) Suppose someone were to apply Newton's method in an attempt to solve $f(x) = 0$, but after choosing an initial guess of x_1, they find that x_2 does not exist. Based on this information, what are the possible values for their initial guess of x_1? (Remember, as with all problems on this exam, you need to justify your result.)
   (c) Approximate $\displaystyle\int_0^2 (f(x) + 3)\,dx$ using a right Riemann sum (use righthand endpoints) with three rectangles of equal width. Please DO NOT simplify your final answer, but it should be in a form that could be directly input into a calculator.

3. (12 points) We want to build a cylindrical tank with a capacity of 24π cubic meters. The cost of the material to make the top and bottom is \$3 per square meter, and the cost of the material to make the side is \$2 per square meter. What dimensions will minimize the cost of the tank? (Be sure to justify that the dimensions you have found minimize the cost.)

   You may find the following two facts helpful:
   - The volume of a cylinder is $V = \pi r^2 h$.
   - The total surface area of a cylinder is $A = 2\pi rh + 2\pi r^2$.

4. (18 points) Consider the function $g(x) = \dfrac{x^{1/3}}{x+2}$, which is referenced in each part of this problem. You may assume without proof that its derivative is given by $g'(x) = \dfrac{2(1-x)}{3x^{2/3}(x+2)^2}$.
   (a) Find all critical numbers of $g(x)$.
   (b) Determine all the local extrema of $g(x)$ and identify each as a local maximum or minimum. Your argument should justify that all of the local extrema have been found. Clearly indicate the x-coordinates where the local extrema occur.
   (c) Determine $\displaystyle\frac{d}{dx}\left(\sec(4x) + \int_1^{\cos x} g(t)\,dt\right)$.

5. (16 pts) Using the grid below, sketch the graph of a **single function**, $y = f(x)$ with each of the following characteristics. (Sketch dashed lines to indicate any asymptotes that are present. The concavity of your graph should be clear.)

f is continuous on its domain:$(-\infty, -2) \cup (-2, \infty)$, $f(-5/2) = 0$

$f(-3/2) = 0$, $f'(x) < 0$ only if x is in $(-\infty, -2) \cup (-2, -3/2)$

$f''(x) > 0$ only if $x > -2$, $\lim_{x \to -\infty} f(x) = 4$

$\lim_{x \to \infty} [f(x) - (4x + 4)] = 0$, $\lim_{x \to -2^-} f(x) = -\infty$