INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from (or giving help to) another person are not permitted while taking the exam. The exam is worth 100 points.

Potentially useful formula: Let u and w denote positive real numbers, then:

$$\log_b(u) = \frac{\log_a(u)}{\log_a(b)} \text{ for } a > 0,\ a \neq 1$$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

iii. WRITE YOUR NAME ON THE NEXT PAGE. JUST BEFORE YOU UPLOAD TO GRADESCOPE WRITE DOWN YOUR UPLOAD TIME ON THE NEXT PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND MOVE TO THE SUBMISSION AREA TO UPLOAD YOUR ANSWERS WITH SUPPORTING WORK TO GRADESCOPE AND HAND IN YOUR PHYSICAL COPY.

v. IF THE UPLOAD AREA IS VERY CROWDED, WE RECOMMEND CHECKING YOUR WORK AND WAITING UNTIL THE AREA IS LESS BUSY. THANK YOU FOR YOUR PATIENCE!

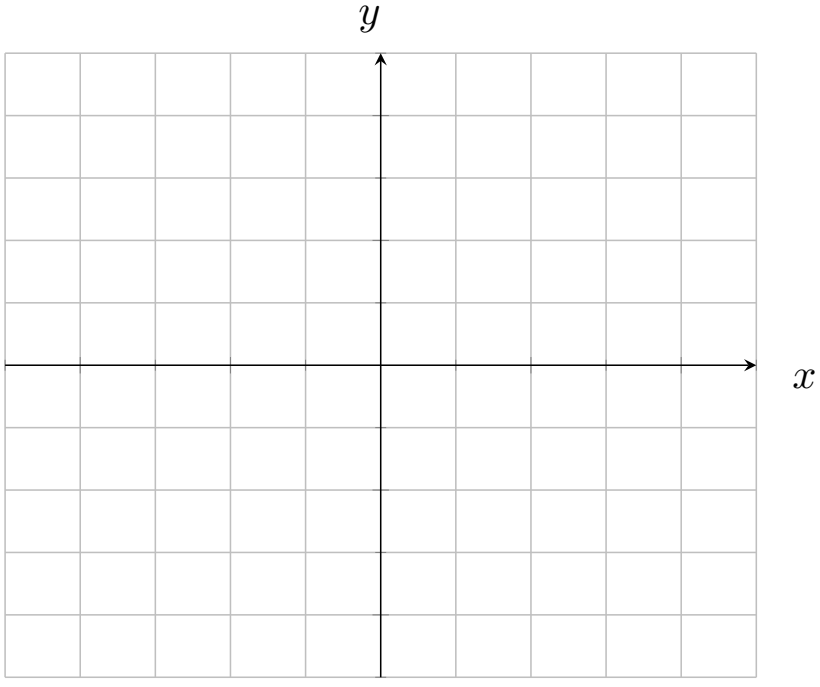
Name: _____

1. Use long division to find both the quotient and remainder when $2x^5 + 2x^4 - x^3 + 5x^2$ is divided by $x^4 - 3x^3$. (4 pts)

2. Answer parts (a) and (b) for the polynomial function, $P(x)$, that satisfies **all** of the given information in i-iv.

   i. The graph has y-intercept $(0, 0)$.
   ii. The graph has end behavior consistent with $y = -x^3$
   iii. The graph crosses at x-intercept $(-2, 0)$
   iv. The graph has no other x-intercepts.

   (a) Sketch the shape of the graph of $P(x)$. **Label** the values of all intercepts on the graph. (4 pts)



   (b) Write down a polynomial $P(x)$ that satisfies all of the given information. (2 pts)

3. Consider the following rational function: $f(x) = \dfrac{2x^4 + 10x^3 + 12x^2}{x^4 + 4x^3 - 5x^2}$. Answer the following: (12 pts)

(a) Find the x-coordinate of any hole(s) of $f(x)$. If there are none write NONE.

(b) Find the y-coordinate of any hole(s) you found in part (a). If there are none write NONE.

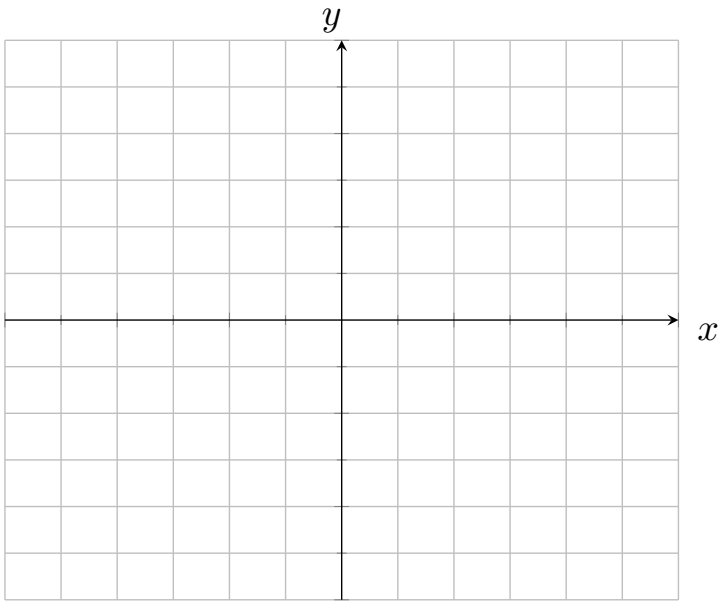
(c) Find all vertical asymptote(s) of $f(x)$. If there are none write NONE.

(d) Determine the end behavior of $f(x)$ and fill in the blanks: $f(x) \to$ ____ as $x \to -\infty$ and $f(x) \to$ ____ as $x \to \infty$.

(e) Find all x-intercept(s) of $f(x)$. If there are none write NONE.

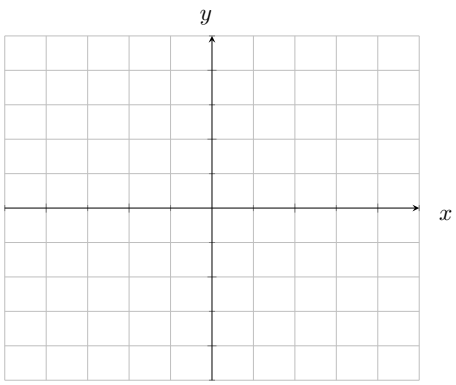
(f) Find the y-intercept. If there is none write NONE.

(g) Sketch the graph of $f(x)$ using parts (a)-(f). **Label** all intercept(s), hole(s), and asymptote(s) as relevant.

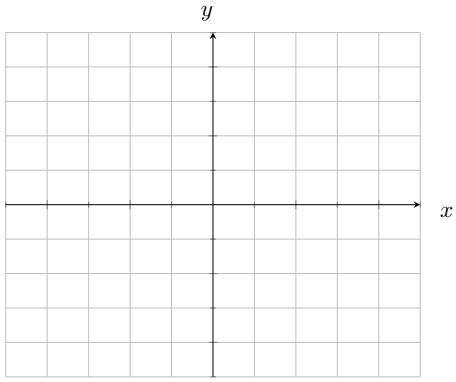


4. For parts (a) and (b) sketch the following graphs and be sure to label any asymptotes and intercepts for each graph:

(a) $g(x) = -e^x$ (3 pts)



(b) $h(x) = \log(x-1)$ (3 pts)



(c) Use your graph in part (a) to fill in the blank: $g(x) \to$ _____ as $x \to -\infty$. (2 pts)

(d) What is the domain of $h(x) = \log(x-1)$, the same function from part (b) (give your answer in interval notation)? (2 pts)

(e) For the function $h(x) = \log(x-1)$ given in part (b) find: $h\left(10^{2x}+1\right)$ (3 pts)

5. The following parts are unrelated.

   (a) Simplify (rewrite without logs): $\log(1) - \log_2(32) - \ln\left(e^{-3}\right) + \log_3(9^x)$ (4 pts)

   (b) Expand to rewrite as a sum/difference of logarithms without any exponents/roots, and simplify as usual: $\log_7\left(\frac{\sqrt{xy}}{7x}\right)$ (4 pts)

6. Solve the following equations for x. If there are no solutions write "no solutions" (be sure to justify answer for full credit).

(a) $4^{5-7x} = 16$ (4pts)

(b) $3^{2x-1} = 2^{x}$ (4 pts)

(c) $\ln\left(10 + x^2\right) = \ln 7 + \ln(x)$ (4pts)

(d) $2\log_2\left(3x^{1/2}\right) = 6$ (4 pts)

7. 340 Siberian cats are introduced onto an island and scientists estimate that the cat population will double every three years. Answer the following questions: (9 pts)

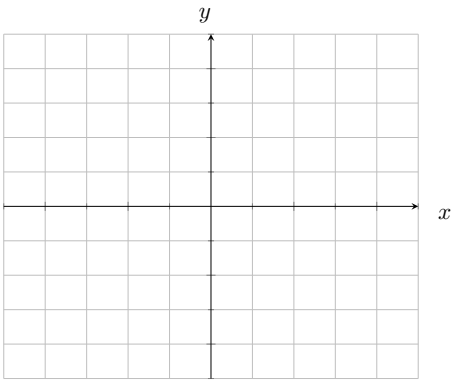
   (a) Find a model of the form $P(t) = P_\circ 2^{t/a}$ that models the population size of cats.

   (b) How many cats are expected to be on the island in 9 years? Leave your answer in exact form (do not attempt to approximate as a decimal value).

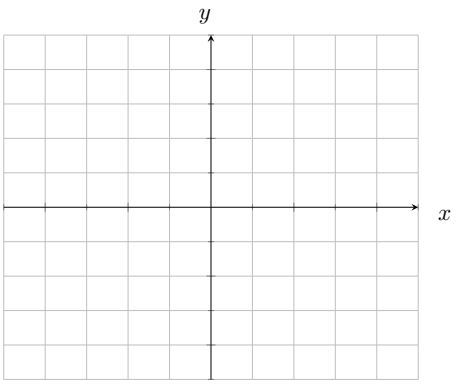
   (c) According to the model found in part (a), how long until the population reaches 2000 cats? Leave your answer in exact form (do not attempt to approximate as a decimal value).

8. Plot the terminal point for each real number on the unit circle (you do not need to find the coordinates of the point).

(a) $t = \dfrac{7\pi}{4}$ (2 pts)



(b) $t = -\dfrac{7\pi}{6}$ (2 pts)



9. Let $\left(\dfrac{1}{4}, y\right)$ be the terminal point on the unit circle corresponding to real number t. Suppose we also know $\sin t < 0$. Use this information to answer the following:

(a) Considering all given information, what quadrant does the terminal point of t lie in? (2 pts)

(b) Find the value for y. (3 pts)

(c) Find $\cos t$ (2 pts)

(d) Find $\sec t$ (2 pts)

EXAM CONTINUES ON BACK

10. Find the exact value of each of the following. If a value does not exist write DNE. (15 pts)

(a) $\sin\left(\frac{\pi}{3}\right)$

(b) $\cos\left(\frac{3\pi}{4}\right)$

(c) $\tan\left(-\frac{2\pi}{3}\right)$

(d) $\sin(0)$

(e) $\csc\left(\frac{5\pi}{6}\right)$

11. Simplify the expression: $e^x\left(e^x + e^{2x}\right) - 2\left(e^x\right)^3$ (4 pts)